The Variance Risk Premium and Investment Uncertainty

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ABSTRACT

This article documents that the variance risk premium in asset returns decreases firms' investment. In our model, the premium increases the value of the real option to postpone an irreversible investment. Empirically, we find support for a negative relationship between variance risk premia and firms' investment as also implied by our model simulations. The relation is more important for investment-grade firms, which tend to have low historical variance but relatively high variance risk premia. Controlling for this premium allows us to reconcile an otherwise surprising pattern across credit ratings: investment rates are higher for speculative-grade than investment-grade firms.

Keywords: Real option, Variance risk premium, Optimal timing, Stochastic variance

JEL classification: D81, G13, G31

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†Direct correspondence to Babak Lotfaliei, San Diego State University, 5500 Campanile Drive, San Diego, CA, USA, 92182-8236. Tel: +1(619) 594-4790. Email: babak.lotfaliei@sdsu.edu. Data and codes to replicate the article's theoretical and empirical results are available at http://www.babakl.com/research.

1 Introduction

We examine the role of the variance risk premium (VRP) in capital budgeting. In a real-option model with stochastic priced variance, we document a negative effect for the VRP of the unlevered assets on the firm's willingness to invest. The simulations of the model also show negative correlation between VRP and firm investments. We find empirical support for this prediction in firm-level data, where we measure a VRP as the difference between the historical and the risk-adjusted (RA) variance.

Time-variation in variances, as well as associated variance risk premia have been extensively documented in the risk management literature.[1] Yet, to date, only time-varying variances, not variance risk premia, have been considered in the real option and capital budgeting literatures (e.g. Bloom, Wright, and Barrero (2016) and Glover and Levine (2015)). A critical factor in capital budgeting decisions is uncertainty of the future project cash flows. The uncertainty is measured by variance or volatility and negatively influences firms' investment activity through at least two channels: a) it changes the discount rate for the cash flows which in turn determines the net present value (NPV) of investments, and b) even with positive NPV, variance influences the value of the real option to postpone. The last channel is stronger when firms face irreversible decisions such as initiating R&D projects (Bloom, 2014). Our contribution is to illustrate and quantify the influence of a variance risk premium (higher risk-adjusted variance) on the firm's optimal investment policy.

When the variance of a project's cash-flow return is both time-varying and loading on economy-wide variance shocks, a variance risk premium arises. Most other risk factors directly influence expected project returns which is a first-moment effect. However, VRP affects the risk-adjusted expected variance of project returns which is a second-moment effect: a risk-averse agent evaluates RA variance to be higher than the historical variance

[1] For example, see Campbell, Giglio, Polk, and Turley (2012), and Bichuch and Sircar (2014) and Garlappi and Yan (2011) for asset pricing and Heston (1993), Christoffersen, Heston, and Jacobs (2013), Cotton, Fouque, Papanicolaou, and Sircar (2004), Fouque, Papanicolaou, Sircar, and Sølna (2011) and van der Ploeg (2006) in the context of derivative pricing.

for cash flows, and adjusts investment decisions accordingly. The real option of waiting to invest has a call-like payoff. The call's value is increasing in RA variance of the underlying state variable. A high VRP increases the RA variance and thus the option's value and makes waiting more attractive. This means that two firms which are identical in all respects but their exposure to economy-wide variance, will differ in their investment policy. The firm with more exposure to systematic variance will, all else equal, have a lower investment rate.

We illustrate this intuition in a preliminary examination of firms across the credit rating spectrum. Rating agencies rate firms based on their historical risk, which provides us with a proxy for grouping firms by the business risk that they face. Between 1998 and 2014, the average investment-grade (IG) firm has about 0.9% net investment rate under relatively favorable investment conditions. For example, it has an operating profit of 10% and an historical asset volatility of 25%; in short, the average investment grade firm is profitable and low risk. The average speculative grade (SG) firm has higher investment rate (2.2%), while its operating environment appears less favorable (for example, the operating profit is lower at 5% and historical asset volatility is higher at 29%). Taking high profitability and low historical asset volatility to be representative of project quality would suggest higher investment (Leahy and Whited, 1996; Bloom, 2009). Thus, firms with high ratings seem to follow a surprisingly conservative investment strategy. However, their volatility loads more on market volatility than SG rated firms. Consistent with this stylized fact, we find that they also have proportionally higher asset variance risk premia than speculative grade firms with historically higher risk. In our sample, IG firms' asset volatility has a 38% correlation with the VIX, compared to 28% for SG firms. We conclude that asset variance risk premia are an important determinant of firms' investment rate, over and above the role played by the level of asset variance itself.

We solve a theoretical model for the real option to invest faced by a firm with priced and stochastic asset variance. In the model, we find that the VRP increases the value of the option to postpone investment and delays a firm's investment in a positive NPV project.

Then, we simulate an economy with a cross section of firms. Each firm is made of multiple projects and faces the decision whether to start or wait and postpone each project which arrives randomly over time. The projects are about the main business of the firm and share similar risk structure, e.g. an oil company that faces similar oil drilling projects. By means of simulations, we observe that the increase in the value of the real option to delay negatively impacts the firms' investment rate.

In the empirical section, we document support for our two hypotheses in data spanning the period 1998 to 2014: a) there is a negative correlation between the investment rate of firms and their VRP and, b) the negative correlation is stronger for the IG firms. In the regressions, we control for other factors such as profitability, growth and historical variance. Investment-grade (IG) firms have relatively favorable investment conditions compared to speculative-grade (SG) firms, such as lower historical variance, lower asset beta, higher profitability, higher Tobin's Q, and lower financial constraints. However, on average, IG firms invest proportionally less than SG firms. Although IG firms have low historical variance and beta, they face high exposure to systematic variance risk. The resulting higher VRP contributes to their conservative investment. We also verify in an out-of-sample test that the variance premium has significant predictive power for firms' investment rates, particularly for investment-grade firms. Our findings are robust as we a) include R&D expenses (which are more likely to be irreversible) in the calculation of the investment rate and b) consider manufacturing-firm sub samples. Both hypotheses are more significant when we include R&D expenses, in particular for manufacturing firms. We thus provide both theoretical support and empirical evidence for the intuition that the VRP reduces firms' investment.

This article's contributions to the real option theory are twofold. First, we show that VRP increases the value of the real option to wait. Grenadier and Malenko (2010) show that agents learn the difference between temporary and permanent shocks to the uncertainty and time the investment accordingly. In this line, Bloom, Wright, and Barrero (2016) find that long-run variance has stronger effect on determining the investment of the firms. In our paper, we extend these studies by including the price of the variance risk. If we shut down the VRP in our model, the model will collapse into a time-varying variance model with a long-run and a short-run variance level. Second, we provide an approximate closed-form solution for the optimal timing of the project startup under stochastic variance. The solution collapses into the traditional Dixit and Pindyck (2012) model when we shut down both the variance time-variation and risk premium.

This article also adds to the empirical literature on investments. Bloom (2014) provides a literature review on the topic. First, in out-of-sample tests, we find that VRP has predictive power for a firm's investments. Second, we also contribute to the literature by considering the cross-section of firms. Most empirical studies focus on the time-series of investment at the aggregate level. For example, Bloom (2009) shows that in the time-series there is negative relation between uncertainty shocks and firms' investments. There are also studies that work with firm-level data, but they do not consider the cross-sectional dispersion of firm characteristics. For example, Bloom, Bond, and Van Reenen (2007) show that uncertainty lowers the effect of demand shocks on firm-level investment. We consider a cross-section of firms defined along credit ratings, as a proxy for historical asset risk. We find that investment-grade firms are more profitable and that their seemingly conservative investment policies can be explained by their higher exposure to variance risk. Therefore, speculative-grade firms are "relatively unprofitable firms that invest aggressively" and "plague the five-factor model in FF" (Fama and French, 2016) because they have less exposure to variance risk.

Finally, our findings also complement the literature on variance risk. Campbell et al. (2012) shows that in macro-level the variance risk has some effect on consumption and asset-pricing factors. McQuade (2012) and Elkamhi, Ericsson, and Jiang (2011) find similar effects in the equity premium and credit spreads of firms. Lotfaliei (2012) reports that the conservative leverage choice by IG firms can be explained by VRP. We show that firms use the investment decision as a means of hedging uncertainty about future systematic asset variance shocks, especially for IG firms.

This paper is organized as follows: Section 2 presents the model, comparative statics, and simulations of the real option to invest with variance risk. Section 3 analyzes the effect of VRP in the time-series and the cross-section of firms and empirically verifies the model implications with robustness checks. Finally, Section 4 concludes the paper.

2 Model

2.1 Setup and Assumptions

Let's consider a firm with an investment opportunity that, whenever started, has initial irreversible fixed cost, Φ, and creates a stream of positive risky cash flows, γ, during its life, ψ. The cash flows follow Geometric Brownian Motion (GBM) with stochastic variance under both physical, P, and RA, Q, measures. Hence, the present value of the future cash flows or the project's value, ν, is also uncertain and follows a similar process before starting the project (see AppendixA for the proof):

$$P\begin{cases}\frac{d\nu}{\nu}=(\mu-\delta)dt+\sqrt{V}dB_1^p\\ dV=\kappa(\theta-V)dt+\sigma\sqrt{V}dB_2^p\\ VRP=\lambda-\kappa\end{cases},Q\begin{cases}\frac{d\nu}{\nu}=(r-\delta)dt+\sqrt{V}dB_1\\ dV=\lambda(\theta^*-V)dt+\sigma\sqrt{V}dB_2,\\ \theta^*=\frac{\kappa\theta}{\lambda}\end{cases}\tag{1}$$

where μ is the drift, δ is the leak in the drift, V is the variance, $\sqrt{V}$ is the volatility (standard deviation), κ is the speeed of mean-reversion, σ is volatility of variance, θ is mean variance (long-run or long-term variance), r is the risk-free rate, θ^* is RA mean variance, and λ is the RA speed of mean-reversion. B_1 and B_2 are independent Brownian motions under Q and B_1^p and B_2^p are independent Brownian motions under physical measure (Appendix B lists the model parameters). The return process has no correlation with variance to keep the model tractable similar to Hull and White (1987). We define strictly positive δ similar to Dixit and Pindyck (2012) to avoid degenerated project and option value. A positive leak also reflects any possible losses in the project's value due to externalities. For example, if the project is about drilling for an oil reserve, the leak is similar to the convenience yield for oil.

The RA and physical variance processes connect based on VRP. In line with Heston (1993), the difference in the mean-reversion speed under RA and physical measures $(\lambda - \kappa)$ corresponds to VRP in this article and mirrors price for variance risk in the project value. The difference is also the variance premium per unit of variance. For example, the instant variance premium is instant variance times the premium per unit of the variance $(VRP_t = VRP.V_t)$. Zero VRP makes RA and physical variances equal but the variance still will be time-varying. Due to variance premium, variance's mean under the physical measure is lower than the RA measure $(\theta^* > \theta)$; a risk-averse agent allocates a higher RA mean to variance because variance has priced risk. We also drop the negative sign and work with its absolute value for simplicity, even though VRP is negative.

There is no agency problem and managers have aligned objectives with shareholders for simplicity. The firm's managers hold the real option to wait and decide when to start the investment project. The project's net present value (NPV) is the project's value less the cost, $\nu - \Phi$. Without the real option to postpone the project, managers only decide once at time zero about the project's fate; they would start (reject) the project, if the expected value is higher (lower) than the costs, $\nu \geq \Phi$ $(\nu < \Phi)$, according to the simple NPV rule. With the real option, however, positive NPV will not necessarily lead to project inception because managers' decision is more complicated due to the uncertainty in value. The managers start the project whenever the expected NPV is maximum. If they start the project pre-maturely, they face a high probability of regretting the decision when the project's income drops below the costs due to non-optimal exercise of the real option. The option to delay the project has no expiry for simplicity because this assumption makes the model time-homogeneous. A real option with expiry does not change the inferences from the model but requires numerical methods rather than a closed-form solution.

In order to start the project at maximum expected NPV, the managers pick a constant boundary, L, at the beginning of each period lasting for T years. The managers update the boundary to a new constant level only at the start of each period. The process continues until the boundary is hit during a period: they commit to start the project when the project value rises above the boundary; once the boundary is hit, the project starts and the real option is exercised. As long as the project value is below the boundary, the managers prefer to wait and delay the project because the real option to wait is more valuable than the immediate NPV. Most of the earlier studies assume a constant boundary. But, we relax constant variance and, subsequently, constant optimal boundary assumptions by considering time-variation for both and also priced variance risk. Only a very large T results in a constant boundary. Allowing the boundary to change at certain points in time creates a degree of freedom over choosing a constant boundary for an infinite horizon. The step-wise change in the boundary and its time-homogeneity also allow us to derive the real option value and optimal exercise policy.

The managers decide about the boundary based on only one state variable, the project's variance. Figure 1 shows the setup of the model. For example, if the variance was constant but the project's value changes, the boundary would remain constant similar to Dixit and Pindyck (2012). The managers do not change the boundary, if the project value changes.[2] This result naturally follows the maximization problem: If the project is not started, the optimal boundary is independent from the current value of the project. The intuition is analogous to the optimal exercise of an American call option; the optimal exercise boundary of the call is independent from the underlying asset's value as long as the call is not exercised.

[Place Figure 1 about here]

Technically, the managers' optimization problem requires optimal boundary, L^*, to satisfy

[2] Although this results is intuitive, we present more details and proof in Online Appendix J.1. For constant variance cases, the result has already been used in the literature. We show that the same result is valid for a more general case.

the smooth-pasting condition:[3]

$$\frac{\partial D}{\partial \nu}|_{(\nu=L^*)} = 0 \tag{2}$$

At state $\nu_0 \in (0, L^*(V_0)]$ and $V_0 \in [0, \infty]$, managers choose optimal L_0^* as a function of the current variance, V_0. Then, at time nT, L_{nT}^* is a function of V_{nT} where $n = \{0, 1, 2, ...\}$. ν is only checked for crossing the boundary L^*. There is time homogeneity at all decision points. For example, if by coincidence $V_{iT} = V_{jT}$, then optimal boundaries are also the same, $L_{iT}^* = L_{jT}^*$, because variance is the only state variable to determine the optimal boundary.

2.2 Real Option Value and Optimal Exercise Boundary

The option's intrinsic value is simply the project's NPV, $\nu - \Phi$. The real-option value is the expected present value of NPV when the boundary is hit. We define $\zeta = ln(\nu/L)$ and forward variance, $\hat{V}$, as the average variance process between time 0 and T. Following Romano and Touzi (1997) and Ito's lemma for ν, we have (see Appendix C):

$$\begin{cases} d\zeta = (r - \delta - \frac{1}{2}\hat{V})dt + \sqrt{\hat{V}}dB \\ \hat{V} = (\int_0^T V_s ds)/T \end{cases} \tag{3}$$

The transformed state variables facilitate pricing the real option. The pricing method is based on the contingent-claim approach. We define the variable $I_{\tau<T}$ as 1, if the start time, τ, is smaller than T and the project starts prior to time T. Then, the real option's value in the RA measure is:

$$D(\zeta_0, \hat{V}_0) = E^Q\left((1 - I_{\tau<T})\left[e^{-rT}D(\zeta_T, \hat{V}_T)\right] + I_{\tau<T}\left[e^{-r\tau}(L - \Phi)\right]\right) \tag{4}$$

The first term in the equation is the discounted value of the real option at time T without exercising the real option. The last term is the discounted value of the real option at exercise.

[3]The first-order condition from the optimization problem, $L^* : \frac{\partial D}{\partial L^*} = 0$, also yields the same result. However, it is easier to work with the smooth-pasting condition.

We condition the option's value on forward variance, which yields (see Appendix D for the details):

$$
\begin{aligned}
&D(\zeta_0, \hat{V}_0) = \Big(L - \Phi\Big) E^Q \Big(e^{H.\zeta_0}\Big) \\
&h = \tfrac{r-\delta-\frac{1}{2}\hat{V}}{\sqrt{\hat{V}}} \qquad H = \tfrac{\sqrt{h^2+2r}-h}{\sqrt{\hat{V}}}
\end{aligned}
\tag{5}
$$

Forward variance is the only random variable within the expectation operator. We use Taylor expansion around the expected forward variance to get an approximate closed-form solution for the option value. The Taylor-expansion method is common in the literature (e.g. see Hull and White (1987) and, recently, Sabanis (2003)). The expansion approximates the expression as (see Appendix E):

$$
E^Q\Big\{e^{(\zeta_0.H)}\Big\} \simeq e^{(\zeta_0.\hat{H})}\Big[1 + \frac{1}{2}(A\zeta_0 + B\zeta_0^2)\Big] \tag{6}
$$

where:

$$
\begin{aligned}
&A = \hat{H}''.E^Q\Big[(\hat{V}_0 - E[\hat{V}_0])^2\Big], \qquad B = \hat{H}'^2.E^Q\Big[(\hat{V}_0 - E[\hat{V}_0])^2\Big] \\
&\hat{H} = H|_{\hat{V}=E^Q[\hat{V}_0]}, \quad \hat{H}' = \tfrac{\partial H}{\partial \hat{V}}|_{\hat{V}=E^Q[\hat{V}_0]}, \quad \hat{H}'' = \tfrac{\partial^2 H}{\partial \hat{V}^2}|_{\hat{V}=E^Q[\hat{V}_0]}
\end{aligned}
\tag{7}
$$

Appendix F presents the formulas for the expected forward variance and variance of forward variance in A and B. We derive the real option value by substituting the expectation expression from Equation 6 into Equation 5:

$$
D(\zeta_0, \hat{V}_0) = (L - \Phi)e^{(\zeta_0.\hat{H})}\Big[1 + \tfrac{1}{2}(A\zeta_0 + B\zeta_0^2)\Big] \tag{8}
$$

If we shut down VRP, the last term in Equation 8 with A and B disappears and the outcome matches the classical model without VRP (see Equation 22 in Appendix D). The formula has some extra terms to adjust for VRP compared to the option's value without VRP.

For the optimal decision to start the project, the smooth-pasting condition results in:

$$
L^* = \Phi.\frac{\hat{H} + \frac{1}{2}A}{\hat{H} + \frac{1}{2}A - 1} \tag{9}
$$

See details in Appendix G. The optimal boundary exists only if $\hat{H} + \frac{1}{2}A$ is larger than 1. If the leak is strictly positive ($\delta > 0$) similar to other classical real option models such as Dixit and Pindyck (2012), this condition is satisfied.

We only use Taylor expansion up to the second moment while it is possible to expand it to higher moments. The second moment is accurate enough to estimate the option's value and it has the advantage to yield a tractable formula. See Appendix H for the approximation accuracy compared with simulations. Fouque, Papanicolaou, and Sircar (2000) and Fouque and Lorig (2011) develop stochastic variance model for option pricing with fast mean-reversion speed. This technique requires numerical calculation of the optimal boundary with the assumption of fast mean-reversion speed. However, we use a technique similar to Tahani (2005) and Sabanis (2003), which yields a tractable closed-form formula.

Similar to the real option value, if we shut down VRP, the optimal boundary is also equal to the boundary without VRP in Dixit and Pindyck (2012). The model in this article adjusts the classical formula for VRP. These adjustments for VRP increases the boundary because A is strictly positive. Since physical variance does not change with or without VRP, but the boundary increases with VRP, the probability to hit the boundary declines under VRP. Therefore, managers wait longer to start the project compared to the model without VRP. Intuitively, for high prices of the variance risk, managers prefer to wait longer and be more cautious to accept projects because RA variance is higher than physical variance. More comparative statics are in the next section.

2.3 Comparative Statics and Hypotheses

The comparative statics show that high VRP increases the real option value and the optimal boundary, which lengthens the wait to start the project. We choose the calibration parameters close to empirical observations. The average historical volatility is about 30% for the project. It matches with the average asset volatility reported for all the rated firms in Schaefer and Strebulaev (2005) and used in several other studies, such as Glover (2016) and Strebulaev (2007). The risk-free rate and the leak are set to 5% and 3% respectively which are close to the empirical averages from Federal reserve and reported asset payout rate for the firms between 1998 and 2014. Volatility of volatility, σ, is 20%, VRP, $|\lambda - \kappa|$, is between 0 and 3, and Mean-reversion speed under physical measure, κ, is 4 similar to reported calibrations results from Elkamhi, Ericsson, and Jiang (2011) and Lotfaliei (2012). Ait-Sahalia and Kimmel (2007) report a variance premium for market equity index close to 5, but firm asset VRP in our calibration is lower due to unlevering and partial exposure of firm to market VRP. The optimal boundary is updated every year, $T = 1$. Project cost is \$100 and it is scalable.

Figure 2 shows the increase in the real option value due to VRP. NPV is value minus costs and the real option to wait hedges paying the irreversible costs of the project. *Ceteris paribus,* VRP increases RA variance. High RA variance increases the RA probability that the project's value becomes smaller than the costs. This is an undesirable outcome for the managers. Hence, risky variance increases the hedge benefits in waiting for higher value and delay paying the project's irreversible costs. When the real option is out-of-the-money, there is no big difference between the real options with and without VRP; starting the project is not even feasible, let alone regretting to start prematurely. The difference between the options is higher for in-the-money real option, $\nu - \Phi > 0$, and when the project is feasible.

From an opportunistic perspective, the real option with VRP is also more valuable than the real option without VRP. Since variance risk is priced, the managers assign a higher RA variance to the project's income compared to historical variance. The real option to wait has a payoff similar to an American call. The real option's value increases with higher RA variance similar to an American call because variance increases the RA probability that the project value will increase. Intuitively, the opportunistic managers will regret the decision to start the project, if the project's value keeps increasing. The high RA probability of increase in the project's value implies high probability of regretting to start the project prematurely. Therefore, the real option value increases with high VRP through higher RA variance.

[Place Figure 2 about here]

Increase in the real option's value for waiting to invest leads to raising the boundary for starting the project. The increase in the option's value makes the managers patiently hold on to their real option which pushes up the boundary. Thus, VRP also increases the boundary and delays the time to start the project. Figure 3 compares the effect of VRP on the boundary for different levels of volatility. The graph also shows that the model with VRP implies much higher inception boundary compared to the model without VRP, $VRP = 0$. For example, a premium of about 3 increases the boundary 50% more than the same model without VRP even for cases with low- volatility. As a result, there is a negative relation between VRP and the investment activity at the firm level as it delays the project start:

[Place Figure 3 about here]

Hypothesis 1. *The variance risk premium(s) has a negative relationship with the firm's investment (H1).*

2.4 Simulation

We simulate the firm behavior to start projects under stochastic variance to verify that a negative relationship is detectable between the firms' investment rate and asset VRP in OLS regressions. *Ceteris paribus*, high VRP raises the boundary to start project but it does not change the physical variance process. Hence, a higher boundary implies lower chance of hitting under the physical measure, which reduces the physical probability that the project is started and leads to lower investment by the firm. The regressions show negative correlation between VRP and investment on simulated firm behavior when we control for other factors such as Tobin's Q and physical variance.

We simulate 1000 firms in 1000 economies over 50 years where we use the last 25 years of data. To replicate the positive NPV project with real option to wait, we assume that the firms have finite individual projects available and decide when to start each project

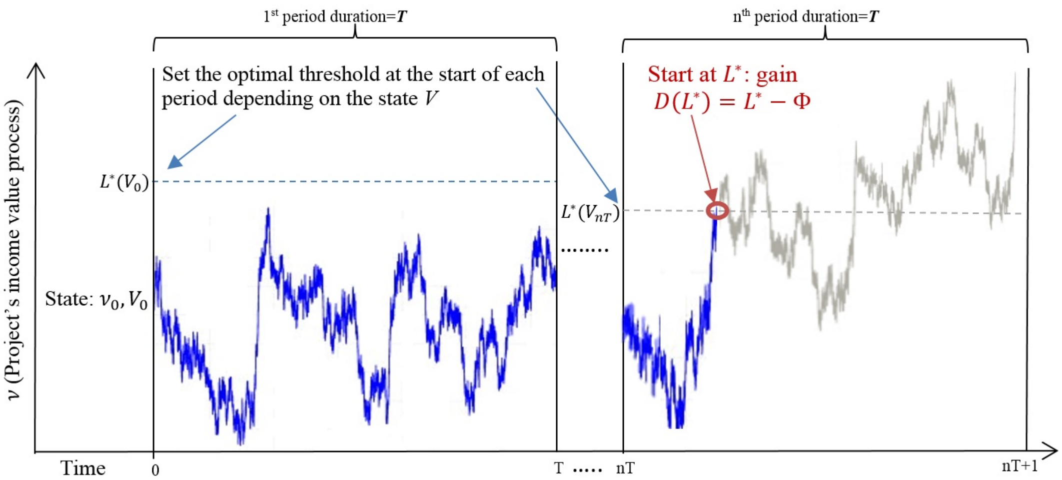


Figure 1 The setup of the model based on the state variables, ν and V

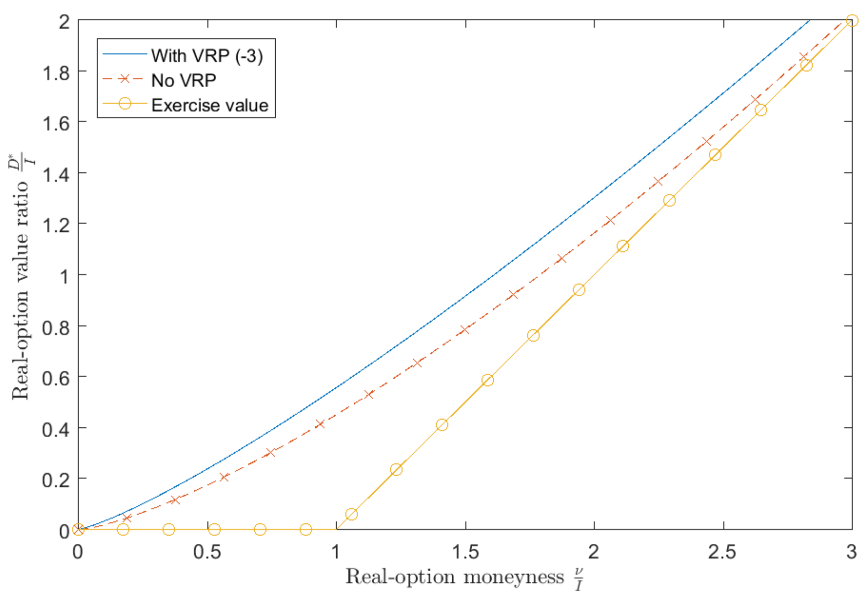


Figure 2 The value of the real option to invest as function of the real-option moneyness.

X-axis shows the real option "moneyness", the ratio of the present value of the incomes to the initial investment cost ($\frac{\nu}{\phi}$). Vertical axis shows the value of the real option relative to the initial investment cost. Mean-reversion speed under physical measure, κ, is 4. The risk-free rate, r, is 5%. The leak value, δ, is 3%. Volatility of variance, σ, is 20%. Initial and mean variances are equal to 9% ($\theta = V_0$) and volatility is 30% ($\sqrt{\theta}$). The decision span, T, is 1 year. Project costs is \$100. Variance risk premium (VRP) increases the value of waiting real option compared to the model without VRP.

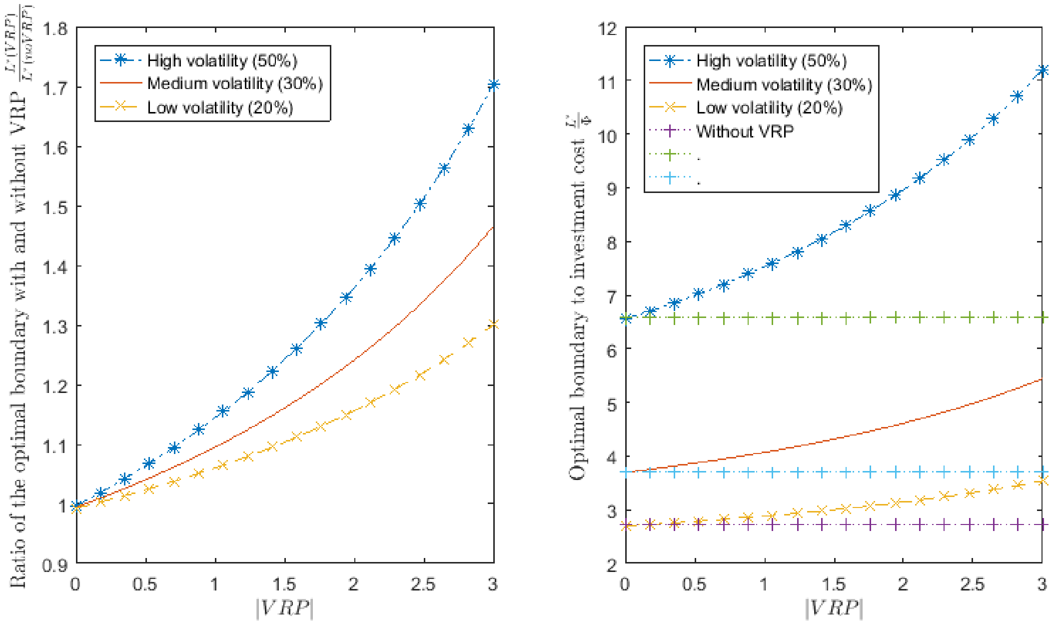


Figure 3 Optimal boundary to start investment as function of the project's variance risk premium (VRP) with different levels of volatility ($\sqrt{V_0}$).
Left graph: X-axis shows VRP. Y-axis shows the ratio of the optimal boundary to the boundary without VRP. Right graph: X-axis shows VRP. Y-axis shows the optimal boundary to investment cost ratio. Each line is for a different volatility. Initial investment cost is \$100. Mean-reversion speed under physical measure, κ, is 4. Risk-free rate, r, is 5% and the leak, δ, is 3%. Volatility of variance, σ, is 20%. Initial and mean variances are equal ($\theta = V_0$).

optimally. For firm i in economy j, new projects arrive according to a Poisson process similar to Kogan and Papanikolaou (2014) with a constant arrival rate, Π_i, specific to the firm. At the start, each project allows the firm to produce products with the exogenous price that follows stochastic variance process. All the parameters of this process such as the mean reversion speeds under physical and RA measures, κ_i and λ_i, are specific to the firm. The present value of all the after-tax cash inflows from product sales at time t, ν_{tij}, also follows the same process. For example, if the company is in oil drilling, the income of every project follows the same process as oil prices. Similar to Equation 1, we have:

$$P \begin{cases} \frac{d\nu}{\nu} = (\mu_i - \delta_i)dt + \sqrt{V}(\rho_i dB_j^p + \sqrt{1-\rho_i^2}dB_{ij}^p), \\ dV = \kappa_i(\theta_i - V)dt + \sigma_i\sqrt{V}dW_{ij}^p, \quad VRP_i = \lambda_i - \kappa_i, \\ \theta_i = \pi_i^2 + (\beta_i\pi_S)^2, \quad \mu_i = r + \beta_i(\mu_j - r), \quad \rho_i = \beta_i\pi_S/\sqrt{\theta_i}, \end{cases} \tag{10}$$

where μ_j is expected market return, β_i is the beta of the value process, π_S^2 and π_i^2 are respectively the variances of the systematic and idiosyncratic shocks to the value process, dB_j^p is the economy-wide shock, dB_{ij}^p is the direct shock to the price process specific to firm i in economy j, dW_{ij}^p is the shock to the variance process, and the shocks are independent.

At the arrival time a, each project requires an initial investment Φ_a following Triangular distribution $Tri(0, \nu_{aij}, \nu_{aij})$.[4] The irreversible investment is paid only at the time, τ, that the firm decides to optimally start the project which creates income $\nu_{\tau ij}$ and the firm locks down on the income. Hence, the project's NPV at start is $\nu_{\tau ij} - \Phi_a$. If the project does not have the real option to wait, the firm starts the project upon arrival because it has positive NPV. If the project has the real option to wait at the arrival, the firm follows the optimal exercise policy as in Equation 9: The firm starts the project if the value is above the boundary, and,otherwise, waits. Hence, each firm has some active projects and a wait list for inactive projects. After the project starts, both its investment and its income depreciate

[4]Where $Tri(min, mid, max) = mid + \sqrt{U(0,1)}[min + U(0,1)(max - min) - mid]$. $U(min, max)$ is the uniform distribution between min and max.

with rate Dep_i and the project only lasts for ψ years where $\psi = Dep_i^{-1}$. In the last year of the project's life, all the remaining investment and income depreciate to zero.

The firms are all-equity and the value of the firms' assets in each year is the present value of all the cash inflows from the active projects plus the sum of the real-option value of the projects in the wait list. For brevity, we ignore the value of the projects yet to come as it is proportional to the price process due to the distribution of the investment costs and does not change the results. Appendix I provides more details about the simulation, such as the parameter values and the value process simulation.

We run two set of regressions on firm-year data in each economy:

$$
\begin{cases}
Inv.Ratio_t = & a_0 + a_1 \times volatility_{t-1} + b_1 \times VRP + a_2 \times Inv.Ratio_{t-1} \\
& +a_3 \times \Delta log(size)_t + a_4 \times Tobin_{t-1} + WN + u \\
Inv.Ratio_t = & a_0 + a_1 \times volatility_{t-1} + b_1 \times VRP \\
& +a_3 \times \Delta log(size)_t + a_4 \times Tobin_{t-1} + WN + \eta_i' + u
\end{cases}
\tag{11}
$$

Volatility is the instantaneous standard deviation of the cash flows. VRP is the difference between each firms' physical and RA mean reversions. Lagged variables are lagged for 1 year. The net capital of the firm is the sum of the remaining capital of the active projects. Tobin's Q is the ratio of total market value of assets to total capital and size is total market value. Change in log-size, natural log of the total market value, represents the firms' growth. Similar to Lang, Ofek, and Stulz (1996), the investment rate is the total investments on the newly started projects less capital depreciation of the active projects in each year deflated by the total capital at the beginning of the year. We also include a white noise variable, WN, in the regressions to compare the results versus a spurious factor. η_i' controls for fixed firm effects. We drop the fixed effect dummy in the first regression in consideration of Nickell (1981)'s critique about the possible bias in the coefficients.[5]

[5] In Online Appendix J.3, these regressions, both on simulations and empirical data, show that the infe-

Table 1 reports the distribution parameters for the regression coefficients across 1000 simulated economies where OLS regressions, on average, show statistically significant negative correlation between VRP and investment[6] Without VRP, the regression coefficients replicate the results from the earlier studies: volatility negatively impacts investment. Growth, measured by change in size, has positive effect. Lagged investment and Tobin's Q also have positive coefficient (Leahy and Whited, 1996; Bloom, Bond, and Van Reenen, 2007; Baum, Caglayan, and Talavera, 2010). The coefficients are significantly different from zero. In regressions with VRP, VRP shows negative impact on investment as suggested in H1, while other coefficients remain similar.

[**Place Table 1 about here**]

3 Empirical Findings

3.1 Data and Preliminary Cross-Sectional Analysis

Between 1998 and 2014, we collect all the firm-year data from Compustat-CRSP merged database and drop financial firms, utility firms (SIC codes 6000-6999 and 4900-4999), and firms with equity market cap, total shares times share value, lower than $9 millions. We match data with Compustat's reported monthly rating of the firms, but we also keep non-rated firms. Investment grade (IG) dummy is 1 if a firm-year has a rating between AAA to BBB. Speculative-grade (SG) firm-years have BB or lower ratings. We also collect historical and option-implied volatilities available from Optionmetrics. Data is limited to the 1998-2014 period because not all the firms have option-trade information.[7] Table 7 in Appendix B

rences about VRP is robust to the bias due to including fixed-effect dummies and lagged dependent variable, which is caused by their correlation (Nickell, 1981). we report the regressions in Table 1 and its empirical twin in Table 3 with both lagged investment and fixed-effect dummy. Lagged investment controls for investment momentum. VRP's negative effect is similar and significant.

[6]The real-option channel is the sole driver behind the relation between asset VRP and investment in this paper. Without the real option to wait, the results are neither significant nor meaningful as presented in Online Appendix J.2.

[7] Nevertheless, in Appendix J.4, we show the trends on investment and investment determinants in the dataset without option-implied volatility, which are similar to trends reported in Table 2 and later in Figure 5.

Table 1 - OLS Regression results on simulated data: Simulation parameters and details are in Appendix I. The tables shows the panel regressions on the investment rate (Inv. ratio) for simulated firm-years in 25 years across 1000 firms. Each coefficient represents the mean across 1000 economies. Standard deviation of the coefficients are in parentheses (coefficient p-values are from their simulation distribution across 1000 economies. p-values test if there are significant number of observations below (above) zero for negative (positive) coefficients:$*p < 0.1, **p < 0.05, ***p < 0.01$). We estimate the regression $Inv.Ratio_t = a_0 + b_1 \times VRP + \text{Control variables} + \eta_i' + u$ with control variables as in Equation 11. Lag variables are lagged for 1 year. Tobin's Q is the ratio of total market value of assets to total capital and size is total market value. Difference in the log size is for 1 year where Log size is the natural log of size. Investment rate is the capital expenditure minus depreciation divided by total capital. Volatility is the instantaneous standard deviation of the cash flows. VRP is the difference between each firms' physical and RA mean reversions. VRP has negative effect on investment based on the sign for b_1 (H1).

Model	(1) Inv. ratio	(2) Inv. ratio	(3) Inv. ratio	(4) Inv. ratio
Lag volatility	-0.080** (0.064)	-0.081** (0.065)	-0.039 (0.175)	-0.039 (0.175)
VRP	- -	-0.016*** (0.009)	- -	-0.244* (0.239)
Lag inv. ratio	0.051** (0.022)	0.051** (0.022)	- -	- -
Diff. Log size	0.577 *** (0.086)	0.577*** (0.086)	0.574*** (0.087)	0.574*** (0.087)
Lag Tobin's Q	0.043*** (0.020)	0.043*** (0.021)	0.068*** (0.031)	0.068*** (0.031)
White noise	0.000 (0.001)	0.000 (0.001)	0.000 (0.001)	0.000 (0.001)
Intercept	-0.107*** (0.068)	-0.076** (0.053)	-0.217** (0.137)	0.329 (0.424)
firm fixed effect	No	No	yes	yes

has the details for all the variables' calculations.

Similar to Denis and Sibilkov (2010), Duchin (2010), Harford, Klasa, and Maxwell (2014), and Aktas, Croci, and Petmezas (2015), the investment ratio is the capital expenditures less depreciation deflated by total book assets. Depreciation is subtracted to measure net investment similar to Lang, Ofek, and Stulz (1996) because some firms have high investments to simply reimburse for depreciation and keep their capital stock at the optimal level. The average investment ratio is in the range reported by Lang, Ofek, and Stulz (1996) and is smaller than the ratio reported in the studies without subtracting the depreciation. We define profitability ratio as net income plus depreciation divided by the book assets to measure operating profitability similar to Cleary (1999). For another profit proxy, we also use cash holdings which is the ratio of the cash to book assets. Cash represents accumulation of the net cash profits of the firm in time minus the payments to stakeholders. Denis and Sibilkov (2010) argue that cash holding allows financially constrained firms to invest more. Although cash has a different role for constrained firms, the rated firms are less likely to face constraints and cash can proxy for accumulated profitability. We also include a proxy for the sales growth of the firm (Bloom, Bond, and Van Reenen, 2007); log change in sales is the first difference in the natural log of the sales.

Following Bloom, Bond, and Van Reenen (2007) and Bloom (2009), we use the annualized 365-day historical equity volatility to represent the historical investment uncertainty in the regressions. In the RA measure, the average over all the strike prices for 365-day call-implied volatility from volatility-surface is the option-implied equity volatility for a day. We fill the missing days with linear interpolation. Then, we take the simple average of equity-implied volatility during the past 365 days before the data date to match the historical volatility calculation based on 365-day historical stock returns.[8] The ratio of the option-implied to historical volatility proxies for the asset VRP. Since the equity historical and implied volatilities are inflated asset volatility by the leverage of the firms (Merton, 1974),

[8]The results are robust using 91-day volatilities which is available upon request.

the ratio cancels out the leverage effect in calculating the VRP proxy. Table 2 shows the descriptive statistics.

The VRP ratio is not exactly equal to the long-run asset-level VRP, but provides a reasonable proxy. Figure 4 shows the time series of the size-weighted VRP proxy and the similar proxy for the market VRP based on VIX index. The market VRP proxy is the ratio of CBOE's VIX index to 30-day historical volatility of S&P-500. While the proxy for the market VRP seems more volatile because of the calculation based on shorter term, this paper's VRP proxy synchronizes with the market's. Both VRP proxies seem low during the crises and they rebound post crises. The trend matches with the intuition about price of variance risk: in normal times, the gap between RA and historical volatilities increases, showing that RA volatility remains relatively high to reflect the risk of the future spikes in volatility. During the crises, historical volatility increases and its gap with RA volatility declines as expected by the market. The figure also reports the size-weighted investment rate of the firm-years which seems to have a negative correlation with both VRP proxies. Anecdotally, the negative correlation supports Hypothesis 1.

[**Place Table 2 about here**]

Insert Figure 4 about here.

We also look at the investment rates and factors across the major risk categories of the firms. The rating companies report the credit score for the firms. Although the credit score serves for credit risk purpose, these ratings also take into account the business risk of the companies and they seem as a valid segmentation for the historical risk of the firms. Therefore, we use the ratings to group the firms based on their historical risk. For example, let's consider asset volatility , the historical standard deviation of the asset returns, which is unlevered historical equity volatility. Historical asset volatility is a measure of business risk (Altman, Resti, and Sironi, 2004). Across the two main rating groups, we observe an increasing trend in historical asset volatility when the rating worsens (see Figure 5a).

Table 2 - Descriptive statistics (1998-2014): Ratings are from monthly Compustat rating by S & P. Table 7 in Appendix B has the details for all the variables' calculations with Compustat codes. Investment ratio is the capital expenditures minus depreciation divided by the total book assets. Cash ratio is the ratio of the cash or equivalents to book assets. Tobin's Q is the ratio of the market to book value of the total assets. Market value of the assets is the sum of equity market cap and the total book liabilities. Profit ratio is the net income plus depreciation divided by the book assets. Volatility is the annualized 365-day historical standard deviation of stock returns from Optionmetrics. Option-implied volatility is the average of 365-day call-implied volatility. VRP proxy is the ratio of option-implied to historical volatilities.

Firm-year group		VRP proxy	Volatility	Tobin's Q	Profit ratio	Cash ratio	Investment ratio
Investment grade	Median	1.023	30.5%	1.607	9.4%	5.6%	0.2%
Obs	Mean	1.034	33.8%	1.886	10.0%	9.1%	0.9%
1,198	Std.	0.193	14.0%	0.977	7.0%	10.1%	3.3%
Speculative grade	Median	0.997	45.7%	1.352	6.8%	5.6%	0.0%
Obs	Mean	1.019	52.7%	1.663	5.3%	10.3%	2.2%
3,491	Std.	0.265	26.2%	1.421	15.1%	13.0%	8.7%
Not rated	Median	1.014	53.1%	1.840	7.6%	23.1%	-0.2%
Obs	Mean	1.052	59.8%	2.673	0.2%	29.5%	0.7%
15,125	Std.	0.285	29.2%	3.667	32.3%	25.1%	7.7%
Overall	Median	1.011	50.1%	1.692	7.6%	16.7%	-0.2%
Obs	Mean	1.045	57.0%	2.447	1.7%	24.9%	1.0%
19,814	Std.	0.277	28.7%	3.293	29.1%	24.2%	7.7%

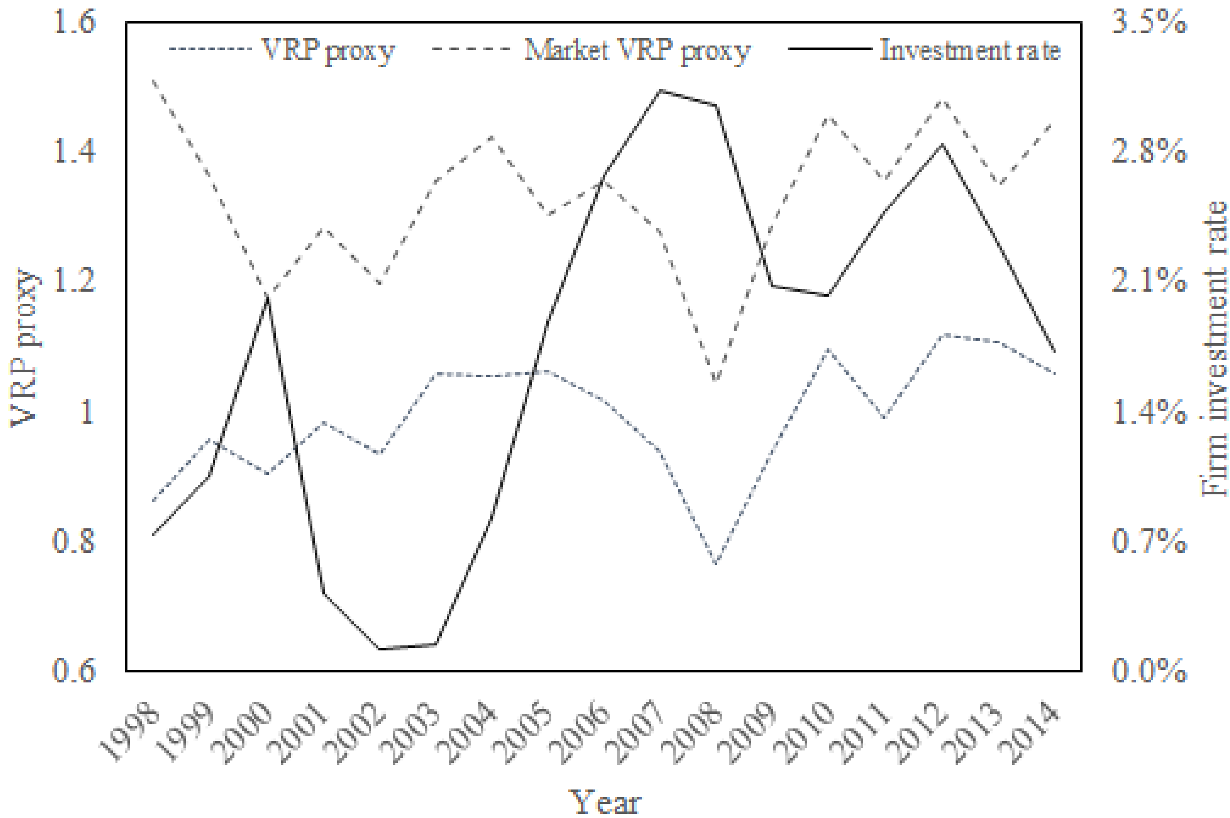


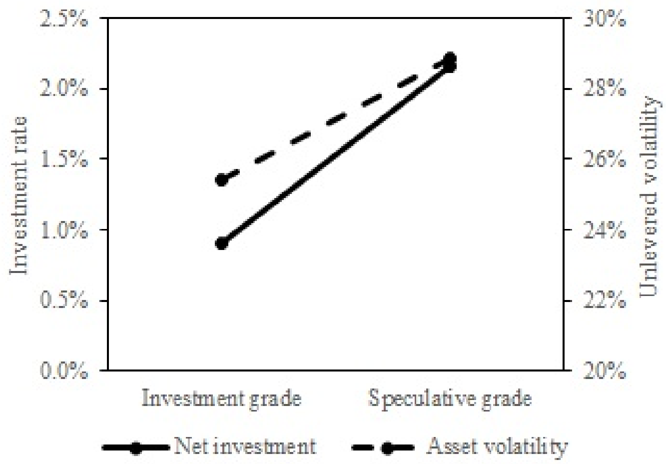
Figure 4 Time series of VRP and firm-level investment.
Left Y-axis shows VRP proxy. Right Y-axis shows the investment rate of the firms. Market VRP proxy is the annual average of all daily values for the ratio of CBOE's VIX to historical 30-day S&P-500 volatility. VRP proxy and investment rate are the size-weighted averages of all the firm-years in each year. Table 7 in Appendix B has the details for VRP proxy and investment rate calculations.

Consequently, the grouping strategy of the firms based on their rating in our preliminary cross-sectional analysis seems to appropriately reflect the business risk.

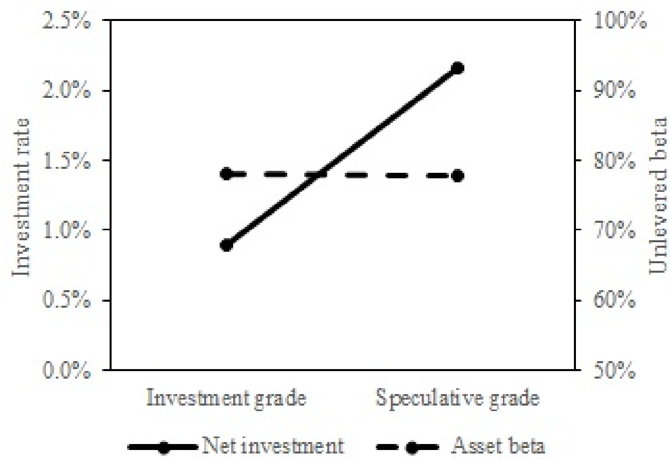
A naive analysis of the traditional investment factors across the ratings show some puzzling investment behavior, which highlights the role of asset VRP. On average, IG firms have lower risk, higher profitability and higher growth potential than the other firms, but IG firms invest more conservatively.[9] All the numbers in the next two figures are the averages of the firm-years for each rating. First, we look at the historical risk. The top two graphs in Figure 5 compares the the investment rate and historical risk trends from asset volatility and asset beta. Asset beta is unlevered equity beta using the leverage ratio of the firm. Both asset-risk trends across ratings match several other papers, such as Schaefer and Strebulaev (2008), Bhamra, Kuehn, and Strebulaev (2010), Elkamhi, Ericsson, and Parsons (2012), and Huang and Huang (2012). Our measure is even more parsimonious because, for example, Huang and Huang (2012) estimate much larger asset beta for SG firms than IG firms using a different method. Nevertheless, in the figures, the average investment increases while historical risk also increases. The investment and risk trends seem counter-intuitive: low-risk firm (IG firms) are expected to invest more, but they invest relatively less than riskier firms (SG firms).

Second, a possible explanation for the investment trend is that the low-risk firms invest less than the riskier firms because they do not have have growth opportunities with high profitability. However, the profitability and growth-potential trends do not support this explanation as in Figure 8c and Figure 8d where growth potential is measured by Tobin's Q. Indeed, both profitability and growth potential have counter-intuitive trends. For IG firms, the profitability is high. Becker and Milbourn (2011) and Ashbaugh-Skaife, Collins, and LaFond (2006) also report positive correlation between rating quality and profitability. Amato and Furfine (2004) find similar trends across ratings for beta, volatility, and profitability as in the top three graphs. But, the IG firms with high profitability are investing more

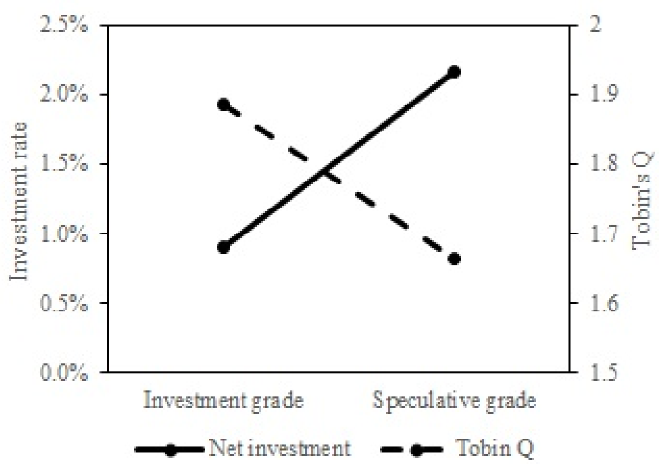
[9] In Online Appendix J.5, we show the trends reported in Figure 5 and Figure 6 are also statistically significant.



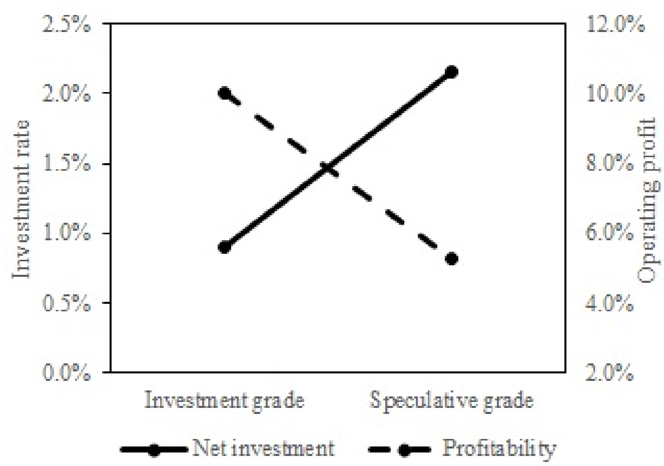
(a) Investment rate and asset volatility



(b) Investment rate and asset beta



(c) Investment rate and Tobin's Q



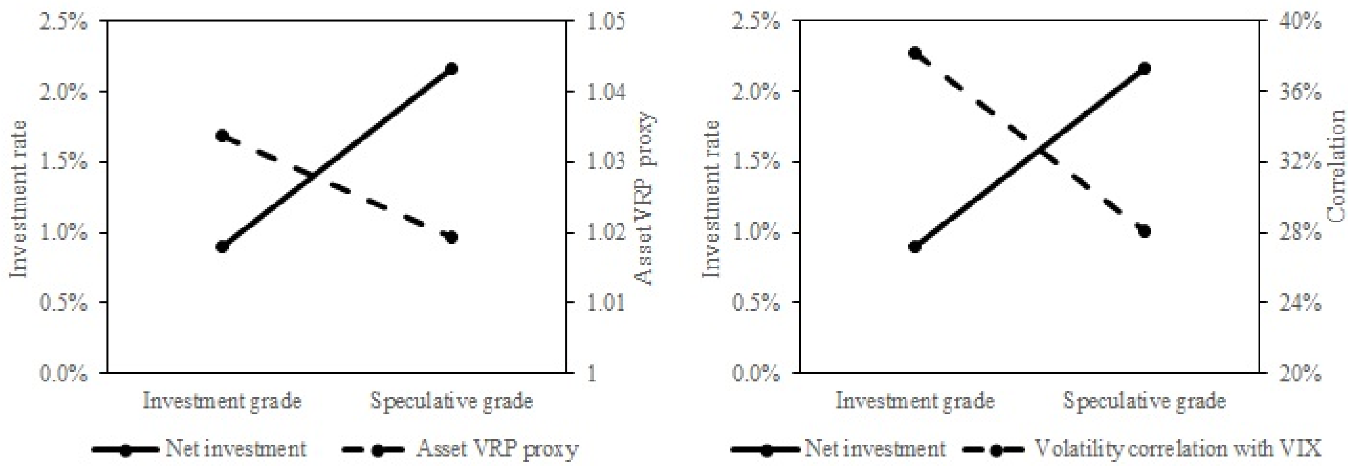
(d) Investment rate and net profit

Figure 5 The trends of the average net-investment rate in contrast with some of the main determinants of the investment across risk grades (1998-2014):
On the X-axis, the rating grades (AAA, AA, A, BBB: Investment Grade (IG). BB, B, CCC and below: Speculative Grade (SG)) are proxy for the risk categories. In each figure, the left Y-axis shows the investment rate. The right Y-axis measures the determinant. Figure 5a shows the average asset return volatility for each category which increases. Figure 5b shows the average asset beta for each category which is flat. Figure 5c shows the average Tobin's Q for each category which decreases. Figure 5d shows the average profitability for each category which decreases. The data for the investment rate, asset volatility, asset beta, profitability and Tobin's Q are the average of firm-years in each grade. The asset return volatility is unlevered equity volatility. The trends in the major investment determinants seem counter-intuitive with respect to the observed investment-rate trend.

conservative than the SG firms with low profitability. The trend in Tobin's Q is similar to Khieu and Pyles (2012)'s study which documents high Tobin's Q for firms with upgraded ratings. Tobin's Q which reflects the growth prospect and investment efficiency is also high for IG firms and does not explain the investment rate. Therefore, the investment trend does not comply with growth potential and profitability trends.

Another alternative to explain the investment trend is possibility of financial constraint and distress for IG firms. Not surprisingly, Elkamhi, Ericsson, and Parsons (2012) report that SG firms are more likely to face financial distress which prevents them from financing their investments in addition to considering the historical risk and profitability factors. In sum, an average IG firm not only has low financial constraints and historical asset risk but also has high profitability and growth opportunities. Seemingly, they invest conservatively and underutilize such favorable conditions compared to SG firms which face worse investment conditions.

Our analysis of VRP provides a potential explanation for the IG firms' conservative investment. In Figure 6, both VRP proxies are higher for top ratings. While IG firms have higher VRP proxy, the difference may look small in Figure 6a based on option-implied volatility. First, the VRP proxy in Figure 6a is based on short-term equity volatility and does not perfectly measure long-term VRP. Hence, this small difference between RA and historical volatility may extrapolate a larger difference in the long-run. Second, we also double check the magnitude of VRP difference in IG and SG firms by looking at their volatility correlation with market volatility in Figure 6b. The average correlation is between VIX and 365-day option-implied volatility of the firm-years for 30 days before the data date in each ratings. The correlation measures the exposure of the firms' volatility to systematic volatility shocks. If a firm's volatility is more exposed to the shocks from market volatility, then the firm also has higher VRP. The IG firms have 1.5 times more exposure to systematic volatility than the SG firms. IG firms' exposure to systematic volatility reduces their investment because they hedge their exposure. While the IG firms have low historical volatility, it is normal that



(a) Investment rate and VRP proxy

(b) Investment rate and exposure to VIX

Figure 6 The trends of the average net-investment rate in contrast with the proxies for asset-level VRP across risk grades.
On the X-axis, the rating grades (AAA, AA, A, BBB: Investment Grade (IG). BB, B, CCC and below: Speculative Grade (SG)) are proxy for the risk categories. The left Y-axis shows the investment rate. The right Y-axis in Figure 6a is the proxy for long-run asset-level VRP. VRP proxy is the ratio of the average 365-day option-implied to historical 365-day equity volatility. The right Y-axis in Figure 6b is the correlation between CBOE's VIX and 365-day option-implied equity volatility for the firm-years during 30 days ending in the data date.

their VRP and RA volatility are high due to this exposure. High RA volatility makes the managers hold on to their valuable real options to wait and more patient about investing. This result provides another anecdotal evidence for Hypothesis 1: IG firms with high VRP invest less. The analysis also leads to the second hypothesis which is specifically about the IG firms:

Insert Figure 5 about here.

Insert Figure 6 about here.

Hypothesis 2. *The variance risk premium(s) has a stronger negative relationship with the firm's investment for the Investment Grade (IG) firms (H2).*

3.2 Hypotheses testing

Following Gulen and Ion (2015), we run the regression below on data:

$$\begin{aligned}
Inv.Ratio_t = \quad & a_0 + a_1 \times volatility_{t-1} + b_1 \times VRP_{t-1} + a_2 \times \Delta log(sales)_t + \\
& a_3 \times Tobin_{t-1} + a_4 \times Cash_{t-1} + a_5 \times Profitability_{t-1} + a_6 \times IGdummy_t \\
+IGdummy_{t-1} \times \quad & \big[c_1 \times volatility_{t-1} + b_2 \times VRP_{t-1} + c_2 \times \Delta log(sales)_t + \\
& c_3 \times Tobin_{t-1} + c_4 \times Cash_{t-1} + c_4 \times Profitability_{t-1}\big] + \eta_t + \eta_i' + u_t + u_i + u
\end{aligned} \tag{12}$$

where u is the error term, η_t and η_i' control for fixed year and firm effects respectively, and u_i and u_t control for clustered errors for the firm and time respectively. Using the method suggested by Petersen (2009), we estimate clustered firm and time errors. Lag variables are lagged for 1 year and difference in the log sales is for 1 year. All other variables are as described in the data section. Firms with high profitability and better investment prospect are more likely to hold cash for future investments and seem to have higher market value with respect to their book assets. Hence, we control for the profitability, and investment prospect with cash holdings and Tobin's Q of the firm. b_1 and b_2 with expected negative signs test the hypotheses, H1 and H2.

Table 3 shows the regression results. R-squared of the regressions is close to the reported values by Gulen and Ion (2015). In the cross-section of the firms, the volatility has negative effect on the investment similar to the findings by Bloom, Bond, and Van Reenen (2007) and Bloom (2009). Tobin's Q has positive effect on the investment in line with Leahy and Whited (1996)'s findings. The cash holdings, profitability and sales growth have positive effect as also reported by Denis and Sibilkov (2010) and Baum, Caglayan, and Talavera (2010).

The regressions support both hypotheses about VRP's role in reducing the investment, especially for the IG firms. VRP has also a negative effect on the investment rate (H1) for all the firms. The negative effect of volatility is slightly weaker for IG firms because of the positive coefficient of the interaction with IG dummy. However, the negative effect of VRP is significantly larger for the IG firms. The increase in VRP's negative effect is almost double for IG firms than the other firms in the sample, which supports H2.

[**Place Table 3 about here**]

3.3 Robustness check

3.3.1 Out-of-sample performance

In order to check the contribution of VRP in the firms' investment decisions, we run out-of-sample tests on the ability of the regressions to predict firm-level investment. The results of the tests show that VRP contributes to the prediction of the firms' investment and its largest contribution is for IG firms. Table 4 presents the results. We divide the sample into two at a breakpoint date. We use the sample before the breakpoint to estimate the regressions. Then, we apply the regression coefficients to the rest of the sample to predict the investment rate for each firm-year. The prediction error is the squared percentage error between estimated and observed investment rate. We drop firm and year dummies in the regressions because they do not apply to firm-years after the break point and replace them with lagged investment. If there are any fixed firm effects, the lagged investment picks the effect. We report root median and mean errors of the regressions.

VRP improves the overall prediction of the firms' investment by about 2%. The contribution is about 9% for IG firms, which is the highest as suggested in Hypothesis 2. The contribution and prediction quality increases as we get distant from the crisis years, which shows stronger effect for VRP in normal times, especially for IG firms. Even when the economy is normal, IG firms face high VRP and hedge for the periods that systematic variance may go up. In sum, VRP has contribution in predicting investment behavior of the firms, especially for IG firms which have more exposure to VRP.

[**Place Table 4 about here**]

Table 3 - Regression results (1998-2014): The tables shows the panel regressions of the investment rate (Inv. ratio) on 19,814 firm-years. We estimate the regression $Inv.Ratio_t = a_0 + b_1 \times VRP_{t-1}$ + Control variables + $b_2 \times IGdummy_{t-1}VRP_{t-1}$ + IG dummy$\times$ Control variables$+\eta_t+\eta_i'+u_t+u_i+u$ with control variables as in Equation 12. Table 7 in Appendix B has the details for all the variables' calculations. IG dummy is 1 if the company has investment grade during the firm-year. Lag variables are lagged for 1 year. Difference in variables is for 1 year. Standard errors are in parentheses (p-values are:$*p<0.1, **p<0.05, ***p<0.01$). VRP has negative effect on investment based on the sign for b_1 (H1). VRP has almost two times stronger negative effect for the investment grade firms (H2) compared to the average firms based on the sign and magnitude for b_2 in Model 6 ($\frac{-0.7\%-0.6\%}{-0.6\%}$).

Model	(1) Inv. ratio	(2) Inv. ratio	(3) Inv. ratio	(4) Inv. ratio	(5) Inv. ratio	(6) Inv. ratio
Lag volatility	-0.0384*** (0.00619)	-0.0384*** (0.00617)	-0.0392*** (0.00607)	-0.0392*** (0.00605)	-0.0330*** (0.00672)	-0.0330*** (0.00671)
Lag VRP	-0.00871*** (0.00265)	-0.00852*** (0.00262)	-0.00915*** (0.00252)	-0.00884*** (0.00248)	-0.00678*** (0.00235)	-0.00649*** (0.00230)
Diff. Log sales	- -	- -	0.00830*** (0.00139)	0.00838*** (0.00142)	0.00851*** (0.00150)	0.00859*** (0.00153)
Lag Tobin's Q	- -	- -	0.00177*** (0.00064)	0.00175*** (0.00064)	0.00146** (0.00061)	0.00145** (0.00061)
Lag Cash ratio	- -	- -	- -	- -	0.0700*** (0.01110)	0.0705*** (0.01120)
Lag Profitability	- -	- -	- -	- -	0.0254*** (0.00708)	0.0254*** (0.00712)
IG dummy	0.00427* (0.00235)	0.00482* (0.00277)	0.00472** (0.00239)	0.00512* (0.00299)	0.00418 (0.00256)	0.00493 (0.00309)
Lag volatility $\times$ IG	- -	0.00871 (0.00874)	- -	0.00398 (0.00809)	- -	0.00705 (0.00770)
Lag VRP $\times$ IG	- -	-0.00371 (0.00329)	- -	-0.00908** (0.00407)	- -	-0.00721* (0.00409)
Lag diff. Log sales $\times$ IG	- -	- -	- -	-0.0122** (0.00557)	- -	-0.0107* (0.00578)
Lag Tobin's Q $\times$ IG	- -	- -	- -	0.00452** (0.00180)	- -	0.00461** (0.00200)
Lag Cash ratio $\times$ IG	- -	- -	- -	- -	- -	-0.0242 (0.01650)
Lag Profitability $\times$ IG	- -	- -	- -	- -	- -	-0.00951 (0.02950)
Intercept	-0.0113** (0.00553)	0.429*** (0.00872)	-0.000188 (0.02960)	-0.000421 (0.02960)	-0.00935 (0.02430)	0.417*** (0.01020)
firm & year fixed effect dummies	yes	yes	yes	yes	yes	yes
firm & year clustered errors	yes	yes	yes	yes	yes	yes
R^2	5.31%	5.32%	6.62%	6.66%	9.83%	9.86%
BIC	-66178.8	-66160.3	-66436.1	-66404.8	-67108.2	-67056.8

Table 4 - Out-of-sample test results: The table shows the marginal contribution of VRP in the out-of-sample (OOS) tests on predicting the investment rate for each firm-year. Limiting the data to the time before the break point, we estimate: $Inv.Ratio_t = a_0 + a_1 \times volatility_{t-1} + b_1 \times VRP_{t-1} + a_2 \times Inv.Ratio_{t-1} + a_3 \times \Delta log(sales)_t + a_4 \times Tobin_{t-1} + a_5 \times CashRatio_{t-1} + a_6 \times Profitability_{t-1} + a_7 \times IGdummy_t + IGdummy_{t-1} \times (c_1 \times volatility_{t-1} + b_2 \times VRP_{t-1} + c_2 \times Inv.Ratio_{t-1} + c_3 \times \Delta log(sales)_t + c_4 \times Tobin_{t-1} + c_5 \times CashRatio_{t-1} + c_6 \times Profitabilityt - 1) + u$. The regression is simple OLS without control for fixed firm and time effects in order to apply the test (Online Appendix J.6 reports the estimated coefficients which also support the hypotheses). Then, we use the coefficients to predict the investment rate of the firm-years after the breakpoint. The error measures the percentage estimation error ($\frac{Observation-Estimated}{Observation}$). IG dummy is 1 if the company has investment grade during the firm-year. Lag variables are lagged for 1 year. Difference in the log sales is for 1 year. Table 7 in Appendix B has the details for all the variables' calculations. The first model has all the variables. The second model has all the variables except VRP and its interaction with IG dummy. The last model has only a constant. The model with VRP reduces errors in predictions mostly for IG firms compared to an average firm.

Sample break point		01, Jan, 2010			01, Jan, 2013		
Regression model		with all variables	all variables without VRP proxy	with only a constant	with all variables	all variables without VRP proxy	with only a constant
Investment grade	Root median squared error	72.5%	75.3%	134.8%	57.9%	67.2%	130.4%
	Root mean squared error	53	67	31	6	7	7
	Obs	515	515	515	226	226	226
Overall	Root median squared error	84.7%	84.9%	125.2%	85.8%	87.5%	127.6%
	Root mean squared error	41	43	38	53	54	61
	Obs	8335	8335	8335	3592	3592	3592

3.3.2 Subsample analysis

The inferences from the earlier regressions are robust in supporting the hypotheses, H1 and H2, when we control for the different measures of investment rate and in the subsample of manufacturing firms (SIC codes between 2000 and 3999). Table 5 presents the results. For some firms, R&D expenses are part of their investments which also seems to be irreversible as well. In order to make sure our results are not only valid for tangible investments, we also include the R&D expenses in the investments. The negative relation between the investments and VRP exists in the sample when we include R&D expenses. While volatility has weaker effect for IG firms, the importance of VRP grows stronger for IG firms because R&D expenses are more likely to be irreversible. The irreversibility contributes to the real option channel in affecting the R&D decisions as argued by Bloom (2007).

We also run the same regressions on the manufacturing-firm subsample to follow some of the earlier studies which only focus on these firms.[10] The negative effect of VRP for IG manufacturing firms is stronger when we include the R&D costs. For all the manufacturing firms, VRP also remains a negative factor for investment decisions. Hence, robustness checks in the subsamples and with including the R&D investments supports the hypotheses presented by the model intuitions.

[**Place Table 5 about here**]

4 Conclusion and Future Research

This article documents that the variance risk premium at the asset level reduces the investments of the firms. Risky and time-varying asset variance increases the hedge value for postponing non-recoverable project costs and waiting to start the projects under more favorable conditions. The managers simply wait for a larger NPV to hedge against the possibility that variance would increase. This relation is also intuitive by considering the

[10] For example, see Bloom (2014), Bloom, Bond, and Van Reenen (2007), and Guiso and Parigi (1999).

Table 5 - Robustness check results with including R&D expenses and in subsamples (1998-2014): The table shows the panel regressions on the investment with controlling for fixed effect and clustered errors. We estimate the regression in Equation 12. The table is similar to Table 3. IG dummy is 1 for the investment grade. Lag variables are lagged for 1 year. Investment rate (Inv. ratio) is the capital expenditure minus depreciation divided by book assets. Investment rate with R&D is the capital expenditure plus R&D expenses minus depreciation divided by book assets which is another proxy for the investment behavior. Table 7 in Appendix B has the details for all the variables' calculations. Control variables are the intercept, the lagged Tobin's Q, Cash ratio, profitability, the difference in the log sales for 1 year, and their interaction with the investment grade dummy. The models 3, 4, 5 and 6 use the subsample of manufacturing firms (SIC codes between 2000 and 3999). Standard errors are in parentheses (p-values are: $*p < 0.1, **p < 0.05, ***p < 0.01$). VRP has negative effect on investment (H1), and VRP has significant and stronger negative effect for the investment grade firms (H2) compared to volatility.

Model	(1) Inv. Ratio with R&D	(2) Inv. Ratio with R&D	(3) Inv. ratio	(4) Inv. ratio	(5) Inv. Ratio with R&D	(6) Inv. Ratio with R&D
Lag volatility	-0.0385***	-0.0386***	-0.0146***	-0.0146***	-0.0349***	-0.0348***
	(0.00826)	(0.00824)	(0.00340)	(0.00337)	(0.01170)	(0.01170)
Lag VRP	-0.0028	-0.00218	-0.00822***	-0.00810***	-0.00422	-0.00326
	(0.00568)	(0.00573)	(0.00278)	(0.00275)	(0.01220)	(0.01220)
IG dummy	0.00467	0.00244	0.003	0.00201	-0.00142	-0.00516
	(0.00347)	(0.00334)	(0.00272)	(0.00237)	(0.00428)	(0.00435)
Lag volatility × IG	-	0.0159	-	0.00291	-	0.0204
	-	(0.01190)	-	(0.00891)	-	(0.01990)
Lag VRP × IG	-	-0.0161***	-	-0.00225	-	-0.0177***
	-	(0.00506)	-	(0.00290)	-	(0.00653)
Control variables	yes	yes	yes	yes	yes	yes
firm & year fixed effect dummies	yes	yes	yes	yes	yes	yes
firm & year clustered errors	yes	yes	yes	yes	yes	yes
R^2	4.35%	4.40%	12.50%	12.60%	4.69%	4.75%
N	19,814	19,814	9,340	9,340	9,340	9,340
Sample	All firms		Manufacturing firms		Manufacturing firms	

analogy between the waiting option and an American call since they have similar payoffs: the exercise boundary for an American call option increases when the variance risk premium is present. The premium implies higher risk-adjusted variance which directly increases the call's value. Similarly, when risk-adjusted variance is higher than the historical variance due to the premium, the managers are more likely to postpone investments. Analogically, the premium increases risk-adjusted asset variance which directly increases the value in waiting.

In the cross-section of the firms, we show that this relation is important in particular for the investment-grade (IG) firms. Without considering VRP, we show that these firms follow a seemingly conservative investment strategy compared to non-IG firms, which is hard to explain with conventional factors for profitability and historical risk. However, these firms face more VRP which explains their conservative behavior. Their high VRP is more likely due to their exposure to systematic variance risk. Therefore, IG firms consider their high exposure to VRP and invest less compared to the other firms on average. In the sample of the firm-years between 1998-2014, we verify that not only VRP proxy has negative relation with investments but also it is stronger for the IG firms.

An extension of this paper can look at the VRP at the aggregate level and its relation with other firm decisions such as hirings. Post economic or financial crises and during normal times, it seems that VRP remains at high levels for a while, even if historical variance gets lower (see for example Bollerslev, Tauchen, and Zhou (2009)). It is possible that VRP, the fear of systematic variance, has contributed to the low recovery after the crises, especially at the firm-level decisions.

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Appendices

A The present value of the project cash flows

The project creates an after-tax stream of positive risky cash flows, γ, during its life, ψ, when it is started. The cash flows follow Geometric Brownian Motion (GBM) with stochastic variance under both physical and RA measures:

$$P\begin{cases}\frac{d\gamma}{\gamma} = udt + \sqrt{V}dB_1^p \\ dV = \kappa(\theta - V)dt + \sigma\sqrt{V}dB_2^p\end{cases} ,Q\begin{cases}\frac{d\gamma}{\gamma} = \alpha dt + \sqrt{V}dB_1 \\ dV = \lambda(\theta^* - V)dt + \sigma\sqrt{V}dB_2\end{cases} \tag{13}$$

where u is the physical drift and α is the RA drift ($\alpha < r$). Since $E^Q(\gamma_s|\gamma_t) = \gamma_t e^{u(s-t)}$, the present value of all the future cash flows, ν, is:

$$\nu_t = E^Q\big(\int_t^{t+\psi} e^{-r(s-t)}\gamma_s ds\big) = \frac{1-e^{-(r-\alpha)\psi}}{r-\alpha}\gamma_t \tag{14}$$

Using Ito's lemma on ν, the value of the project follows:

$$P\begin{cases}\frac{d\nu}{\nu} = udt + \sqrt{V}dB_1^p \\ dV = \kappa(\theta - V)dt + \sigma\sqrt{V}dB_2^p\end{cases} ,Q\begin{cases}\frac{d\nu}{\nu} = \alpha dt + \sqrt{V}dB_1 \\ dV = \lambda(\theta^* - V)dt + \sigma\sqrt{V}dB_2,\end{cases} \tag{15}$$

B List of the Theoretical and Empirical Parameters

[Place Table 6 about here]

[Place Table 7 about here]

C Change of Variable

Define $\zeta = ln(\nu/L)$. With Ito's lemma, the process for ζ is:

$$d\zeta = (r - \frac{1}{2}V)dt + \sqrt{V}dB \tag{16}$$

Integrating the process for the solution of the differential equation yields:

$$\zeta_T - \zeta_0 = rT - \frac{1}{2}\int_0^T V dt + \int_0^T \sqrt{V}dB \tag{17}$$

Let's define $\hat{V} = (\int_0^T V_s ds)/T$ which transfers the equation into:

$$\zeta_T - \zeta_0 = (r - \frac{1}{2}\hat{V})T + \int_0^T \sqrt{V}dB \tag{18}$$

ζ_T follows a Brownian motion with mean $\zeta_0 + (r - \frac{1}{2}\hat{V})T$. Its variance is equal to $\hat{V}T$. Again, we use Ito's lemma on the equation 18 and get:

$$d\zeta = (r - \frac{1}{2}\hat{V})dt + \sqrt{\hat{V}}dB \tag{19}$$

D Derivation of Real Option Formula

D.1 Constant variance

Since we will use conditioning on forward variance, we first value the option when forward variance is constant. With constant forward variance, ζ follows a process with constant variance and the the boundary is also constant. Then, the value of the real option is:

$$\begin{aligned} &D(\zeta_0) = E^Q\left((1 - I_{\tau<T})\left[e^{-rT}D(\zeta_T)\right] + I_{\tau<T}\left[e^{-r\tau}(L - \Phi)\right]\right), \\ &d\zeta/\zeta = (r - \delta)dt + \sqrt{\epsilon}dB_s \end{aligned} \tag{20}$$

where ϵ is the *constant* forward variance. With constant variance, the option's value satisfies the following PDE and the Dirichlet conditions:

$$\frac{1}{2}\epsilon\nu^2 D_{\nu\nu} + (r-\delta)\nu D_\nu - rD = 0, D(L) = L - \Phi, D(0) = 0 \tag{21}$$

where D_ν is the partial derivative of D with respect to ν. The solution and the value of the real option is:

$$\begin{aligned} &D(\nu) = \Big(L - \Phi\Big)e^{H.\zeta} \\ &h = \tfrac{r-\delta-\frac{1}{2}\epsilon}{\sqrt{\epsilon}} \qquad H = \tfrac{\sqrt{h^2+2r)}-h}{\sqrt{\epsilon}} \end{aligned} \tag{22}$$

With constant variance, this result is similar to the option's value from Dixit and Pindyck (2012).

D.2 Stochastic variance

We will condition on forward variance to derive the real option value. We restate Equation 4:

$$D(\zeta_0, \hat{V}_0) = E^Q\Big((1 - I_{\tau<T})\Big[e^{-rT}D(\zeta_T, \hat{V}_T)\Big] + I_{\tau<T}\Big[e^{-r\tau}(L - \Phi)\Big]\Big) \tag{23}$$

Then, we condition on forward variance:

$$D(\zeta_0, \hat{V}_0) = E^Q\Big(E\Big\{\Big[(1 - I_{\tau<T})e^{-rT}D(\zeta_T, \hat{V}_T)\Big] + \Big[I_{\tau<T}e^{-r\tau}(L - \Phi)\Big]|\hat{V}\Big\}\Big) \tag{24}$$

The term conditioned on forward variance has a solution as in Equation 22. Plugging the solution for constant variance in the conditioned term and replacing ϵ with $\hat{V}$ yield:

$$D(\zeta_0, \hat{V}_0) = (L - \Phi)E^Q\left(e^{H.\zeta_0}\right) \tag{25}$$

E Taylor Expansion

Any function's expected value can be approximated by Taylor expansion around forward variance's expected value:

$$
\begin{aligned}
E[F(\zeta_0,\hat{V}_0)] \; &\simeq E\Big[F\Big(\zeta_0,E[\hat{V}_0]\Big)+F'\Big(\zeta_0,E[\hat{V}_0]\Big).\Big(\hat{V}_0-E[\hat{V}_0]\Big) \\
&+\tfrac{1}{2}F''\Big(\zeta_0,E[\hat{V}_0]\Big).\Big(\hat{V}_0-E[\hat{V}_0]\Big)^2\Big] \\
&= F\Big(\zeta_0,E[\hat{V}_0]\Big)+\tfrac{1}{2}F''\Big(\zeta_0,E[\hat{V}_0]\Big).E\Big[\Big(\hat{V}_0-E[\hat{V}_0]\Big)^2\Big]
\end{aligned}
\tag{26}
$$

From Equation 5, we replace $F(\zeta_0,\hat{V}_0)=e^{(\zeta_0.H)}$. Since the function is exponential, deriving its higher order derivatives is simple:

$$
\begin{aligned}
F' &= e^{\zeta_0.H}\zeta_0.H' \\
F" &= e^{\zeta_0.H}\Big((\zeta_0.H')^2+\zeta_0.H''\Big)
\end{aligned}
\tag{27}
$$

F MGF and Variance Moments

We use the Moment Generating Function (MGF) to calculate the moments of forward variance:

$$
\begin{aligned}
&E^Q\Big([\hat{V}_0-E(\hat{V}_0)]^2\Big)=V_0M_2+N_2, \qquad E^Q[\hat{V}_0]=V_0M_1+N_1 \\
&M_1=\tfrac{1-exp(-T\lambda)}{T\lambda}, \qquad M_2=\tfrac{\sigma^2}{T^2\lambda^3}\Big[1-2T\lambda\, exp(-T\lambda)-exp(-2T\lambda)\Big] \\
&N_1=\theta^*\Big[1-M_1\Big], \qquad N_2=\theta^*\Big(\tfrac{\sigma^2}{T\lambda^2}\Big[1+exp(-T\lambda)-2M_1\Big]-\tfrac{1}{2}M_2\Big)
\end{aligned}
\tag{28}
$$

The variance for forward variance is:

$$
E^Q[(\hat{V}_0-E[\hat{V}_0])^2]=E^Q[\hat{V}_0^2]-E^Q[\hat{V}_0]^2 \tag{29}
$$

Thus, we have to derive the two moments of forward variance, $\hat{V}$. We follow Tahani (2005) and get the Moment Generating Function (MGF) using the Feynman-Kac approach. The

MGF for the average variance is:

$$MGF(\hat{V},x) = E[exp(x\hat{V})] = E[exp(x\int_0^T V_s ds)] = F(V_0,x) \tag{30}$$

The expected value follows Feynman-Kac PDE as function of variance, V (for simplicity I drop the index "0"):

$$\frac{\partial F}{\partial t} + (\kappa\theta - \lambda V)\frac{\partial F}{\partial V} + \frac{1}{2}\sigma^2 V\frac{\partial F}{\partial V} + xVF = 0 \tag{31}$$

The solution and MGF for $\hat{V}$ is log-linear in V:

$$\begin{aligned}
&MGF(\hat{V},x) = exp(V_0 M(T,x) + N(T,x)) \\
&M(T,x) = \frac{2x(1-e^{-\omega T})}{T(\lambda+\omega)(1+\phi e^{-\omega T})} \\
&N(T,x) = \tfrac{-2\kappa\theta}{\sigma^2}[log(\tfrac{1+\phi e^{-\omega T}}{1+\phi}) - \tfrac{(\omega-\lambda)T}{2}] \\
&\omega = \sqrt{\lambda^2 - \tfrac{2x\sigma^2}{T}} \quad ; \quad \phi = \tfrac{\omega-\lambda}{\omega+\lambda}
\end{aligned} \tag{32}$$

Both moments are calculated as $E^Q[\hat{V}_0] = MGF'|_{x=0}$ and $E^Q[\hat{V}_0^2] = MGF''|_{x=0}$.

G Optimal Investment Boundary

From Equation 8, we drop "0" index and we have:

$$D(\zeta,\hat{V}) = (L-\Phi)exp\Big(\zeta.\hat{H}\Big)\Big[1 + \frac{1}{2}(A\zeta + B\zeta^2)\Big] \tag{33}$$

The smooth-pasting condition, $\frac{\partial D}{\partial \nu}|_{(\nu=L^*)} = 0$, has the following form on this function:

$$\begin{aligned}
&\tfrac{\partial D}{\partial \nu} = (L-\Phi)\Big[(1+Z)\tfrac{\hat{H}}{\nu}(\tfrac{\nu}{L})^{\hat{H}-1} + \tfrac{1}{2}(\tfrac{\nu}{L})^{\hat{H}}(\tfrac{A}{\nu} + \tfrac{2B}{\nu}ln(\tfrac{\nu}{L}))\Big] \\
&\tfrac{\partial D}{\partial \nu}|_{(\nu=L^*)} = (L^*-\Phi)[\tfrac{\hat{H}}{L^*} + \tfrac{1}{2}\tfrac{A}{L^*})] \\
&Z = A\,ln(\tfrac{\nu}{L}) + B\,ln(\tfrac{\nu}{L})^2
\end{aligned} \tag{34}$$

It is straight forward to see that L^* is equal to:

$$L^* = \Phi . \frac{\hat{H} + \frac{1}{2}A}{\hat{H} + \frac{1}{2}A - 1} \tag{35}$$

The same expression can be derived through setting $\frac{\partial D}{\partial L} = 0$.

H Approximation Accuracy

The approximate closed-form formula for the real option with VRP is close to the actual value. In this section, we show that the average approximation error is less than 3% in most of the cases to support using the Taylor expansion. Although the Taylor expansion does not offer a precise formula, but the value is reasonably close while it also enables deriving a closed-form formula.

Figure 7 compares the simulated values of the real options and the value from the closed-form formula. We check the accuracy of the closed-form formula over two important factors: volatility, and VRP. Values for other parameters are similarly close.

[Place Figure 7 about here]

We simulate the perpetual option over 100 years because it is not possible to completely simulate a perpetual option. For the simulation, we draw 100,000 paths of both income value and variance with 3.6 day steps (1/100 of a year). Random numbers are antithetic to converge faster. If the income value hits the optimal boundary, we discount the exercise value. Otherwise, we assume that if the option is not exercised by the end of the 100th year, the option follows the closed form formulas with constant variance equal to the mean variance without VRP. It is more accurate because an exact value formula exists for the case without VRP. Although this assumption seems to violate the assumption about VRP, it does not significantly affect the average real option value since the values of the options are discounted over 100 years ($exp(-5\% \times 100) = 0.6\%$). At the end of simulation, the real option value is the average present value of the options in each path.

I More simulation details

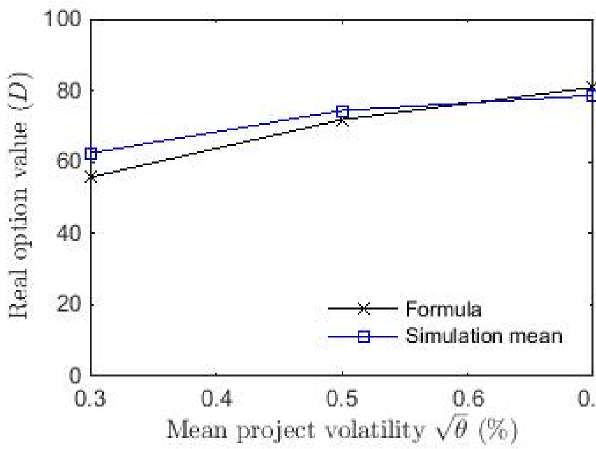
The price process is time continuous, but the simulation is time discrete:

$$\begin{aligned} \nu_{tij} &= \nu_{(t-\Delta t)ij} exp\big[(\mu_i - \delta_j - \tfrac{V_{(t-\Delta t)ij}}{2})\Delta t + \sqrt{\Delta t}(\sqrt{V_{(t-\Delta t)ij}}[\rho_i z_t^S + \sqrt{1-\rho_i^2} z_t^O])\big] \\ V_{tij} &= \| V_{(t-\Delta t)ij} + \big[\kappa_i(\theta_i - V_{(t-\Delta t)ij})\big]\Delta t + \sqrt{\Delta t}\big[\sigma_i \sqrt{V_{(t-\Delta t)ij}}\big] z_t^V \| \end{aligned} \tag{36}$$

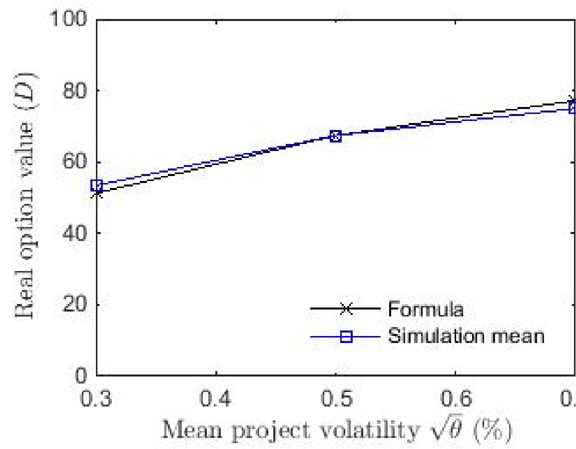
where z^O, z^S and z^V are respectively standard normal shocks to the firm, the economy, and variance. Δt is one month (1/12). The absolute value ensures positive variance. When the price process falls below 1, we replace the firm with a new firm which is the same as the firm in initial simulation point to avoid very small values. In order to save memory, if the NPV of any project in the wait list goes below 0 at any point in time before its start, the firm drops the project. In order to save simulation time, if the optimal start time is during the fiscal years, then the start time is shifted into the beginning of the next financial year with the same NPV at the original start time.

Table 8 presents all the parameter values for the simulation. At the beginning, all the firms start with one project, capital of \$100 and market value of \$100. We only pick the last 25 years of the simulation to reduce the effect of the initial points and to simulate data similar to the empirical section which is limited to 20 years. The project arrival distribution parameters are borrowed from Kogan and Papanikolaou (2014). Following their procedure, we also drop any observations with no active projects. The average depreciation is 5% close to Nadiri and Prucha (1996) and implies life average of 20 years for the projects. The probability of having the real option to wait for a project is set to 80%[11] which puts the simulation investment rate and Tobin's Q in the range of the empirical values. The simulation range and distributions for asset beta, idiosyncratic variance, systematic variance, and market premium are similar to

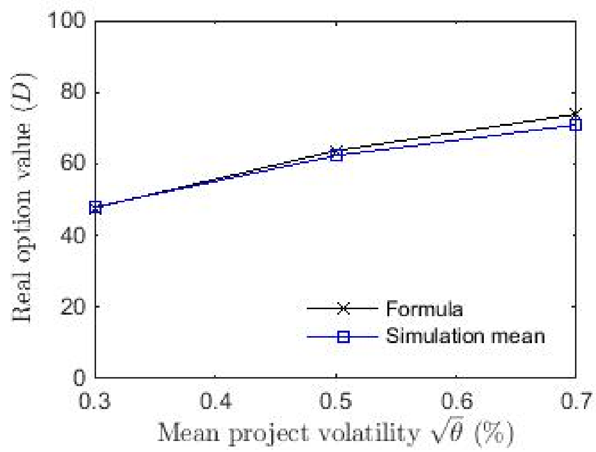
[11] We also report the simulation results in Online Appendix J.2 where the probability is zero. The regression results without real option show that VRP and volatility have no effect on investment and the real option is the only dynamic in the simulations.



(a) $|VRP| = 3$



(b) $|VRP| = 2$



(c) $|VRP| = 1$

Figure 7 Approximation accuracy of the closed-form formula in comparison with the simulated real option values.

We compare the real option value from simulation with the closed-form formula. The average error in Figure (a) is 3.8%. The average error in Figure (b) is 0.4%. The average error in Figure (c) is 2%. Mean-reversion speed under physical measure, κ, is 4. Risk-free rate, r, is 5% and the leak, δ, is 3%. Volatility of variance, σ, is 20%. Initial and mean variances are equal ($\theta = V_0$). Project cost is \$100 and the real option is at the money. The real option from simulations is the average discounted option payoff following the optimal policy over the simulation of 100,000 paths. Each path is simulated for 100 years and time steps are every 1/100 of a year. The decision period is 1 year and the boundary to exercise the option is chosen optimally.

Strebulaev (2007), which is based on several empirical studies. Average risk-free rate and leak in the drift, or the payout rate, match the averages of empirical values between 1998 and 2014.

[**Place Table 8 about here**]

Table 6 -The list of the variables and parameters used in the theoretical model

γ	the project's future cash flow or income,	ψ	the project's life after start
ν	the present value of the project's future cash flows or income, (state variable)	Φ	the project's startup cost
V	the project's income variance, (state variable)	$\sqrt{V}$	the project's income volatility (standard deviation)
κ	the mean-reversion speed under physical measure	λ	the Risk-adjusted (RA) speed of mean-reversion
$\lambda - \kappa$	the Variance Risk Premium (VRP) per one unit of variance	σ	variance volatility
r	the risk-free rate	μ	the return drift under physical measure P
θ	mean variance (long-run or long-term variance)	θ^*	Risk-adjusted (RA) mean variance
L^*	the optimal boundary to start	T	the period length for following optimal exercise policy
B_1^p and B_2^p	independent Brownian motions under physical measure P	B_1 and B_2	independent Brownian motions under Risk-adjusted (RA) measure Q
$\hat{V}$	forward variance	δ	value leak for the project
D	the real option value which is the *expected* NPV of the project at exercise,	$I_{\tau<T}$	is 1, if exercising the option happens prior to time T

Table 7 -The list of the empirical parameters

Parameter	Calculation method	Source
Investment rate	ratio of the capital expenditure minus the depreciation to the book assets (CAPX-DP)$_t$/AT$_t$	COMPUSTAT
Investment rate with R&D expenses	ratio of the capital expenditure and R&D costs minus the depreciation to the book assets (CAPX+XRD-DP)$_t$/AT$_t$	COMPUSTAT
Profitability ratio	the operating profit divided by book assets (NI+DP)$_t$ /AT$_t$	COMPUSTAT
Equity market cap	common shares times the share price (CSHO × PRCC_C)	COMPUSTAT
Tobin's Q	market to book value of the assets (Market cap + LT)$_t$/AT$_t$	COMPUSTAT
Equity beta	Cov(X,Y)/VAR(X), X={S&P return for 250 days}, Y={stock return for 250 days}	CRSP
Market leverage	total debt divided by debt plus equity market value (DLTT+DLC)$_t$/(DLTT+DLC+Market cap)$_t$	COMPUSTAT
Option-implied equity volatility	daily average of 365-day call-implied volatility from the volatility surface for all the strike prices	Optionmetrics
Historical equity volatility	365-day historical standard deviation of the equity returns	Optionmetrics
VRP proxy	365-day average of the option-implied equity volatility divided by historical equity volatility	Optionmetrics
Asset volatility	unlevered historical equity volatility (Equity historical volatility$_t$ ×(1-market leverage$_t$))	COMPUSTAT, Optionmetrics
Asset beta	unlevered equity beta (Equity beta × (1-market leverage))	COMPUSTAT, CRSP
Market VRP proxy	VIX divided by 30-day historical S&P-500 return standard deviation	CBOE, Optionmetrics
Log change in sales	natural log of total revenues in each year relative to the last year $ln(SALE_t/SALE_{t-1})$	COMPUSTAT
Cash ratio	total cash and equivalents to the book assets (CHE$_t$/AT$_t$)	COMPUSTAT
Systematic volatility exposure	Correlation(X,Y), X={VIX for 30 days}, Y={option-implied equity volatility for 30 days}	CBOE, Optionmetrics

Table 8 - Parameter values in the simulation for the model with stochastic variance: ν_{0ij} is the initial present value for the cashflows at time 0 for firm i in economy j. Φ_{0ij} is the initial capital for the sole project held by the firms at time 0. ϕ_i correlates the value shocks with the economy-wide shocks and depends on parameters such as asset beta. π_S^2 and π_i^2 are respectively the variances of the systematic and idiosyncratic shocks to the value process. θ_i is mean variance. κ_i and λ_i are respectively the physical and RA mean-reversion speeds. σ_i is the standard deviation for the total variance of the shocks. T is the period length for following optimal exercise policy. δ_i is the leak. r is the risk-free rate. $\mu_j - r$ is the market risk premium. Pr is the probability of having the real option to wait for a project. Π_i is the project arrival rate per year for Poisson distribution. Dep_i is the project depreciation rate upon start. ψ_i is the life of the project after start. Inv. ratio is the investment rate and Tobin's Q is the market to book ratio of the assets value as described in Section 2.4. $U(min, max)$ is the uniform distribution between min and max. $Tri(min, mid, max) = mid + \sqrt{U(0,1)}[min + U(0,1)(max - min) - mid]$ is Triangular distribution.

Parameter	Distribution	Mean	Std
ν_{0ij}, Φ_{0ij}	constant	\$100	-
Asset $\beta_i > 0$	normal	98.8%	39.7%
π_i	$z_0 + z_1\chi^2(z_3)$	21.9%	16.3%
	$z_0 = 0.05, z_1 = 1/12, z_3 = 2$		
π_S	constant	14.8%	-
$\sqrt{\theta_i}$	$\sqrt{\pi_i^2 + (\beta_i\pi_S)^2}$	27.8%	14.8%
ρ_i	$\beta_i\pi_S/\sqrt{\theta_i}$	0.6	24.2%
κ_i	$Tri(3, 4, 5)$	4	39.5%
λ_i	$Tri(1, 2, 3)$	2	41.2%
σ_i	$Tri(0.1, 0.2, 0.3)$	0.2	4.2%
T	constant	1	-
δ	$U(0.02, 0.04)$	3.0%	0.58%
r	constant	5.0%	-
$\mu_j - r$	constant	7%	-
μ_i	$r + \beta_i(\mu_j - r)$	11.9%	2.8%
Pr	constant	80%	-
Π_i	$k_0 - k_1 ln(U(0,1))$	4	2
	$k_0 = 2.0, \quad k_1 = 2.0$		
Dep_i	$m_0 - m_1 ln(U(0,1))$	5.0%	2.0%
	$m_0 = 0.03, \quad m_1 = 0.02$		
ψ_i	$1/Dep_i$	22	7
Inv. ratio	from simulation	7.8%	30.9%
Tobin's Q	from simulation	3.93	2.43

J Online Appendices

J.1 Proof for Proposition 1

Assumption 1. *The real option's value is increasingly monotonic in the project's value* $(\nu_1 < \nu_2 \rightarrow D(\nu_1) \leq D(\nu_2))$

If $D(\nu_1)$ was larger then $D(\nu_2)$, the managers would destruct some of the project's incomes at ν_2 and the value at ν_2 becomes equal to the value at ν_1 after destruction. Thus, the real-option value at ν_2 is as large as the value at ν_1.

Assumption 2. *The real-option value, D, is continuous in the project's value, ν.*

PROPOSITION 1. *The optimal boundary, L^*, does not depend on the project's value, ν, when the project's value is below the boundary.*

Proof. For two different income values, we show that the optimal boundary is the same and the boundary is independent from the value. The real option is already exercised when the income values are above the optimal boundary. We only focus on the cases which the project value is below the boundary where $\nu \leq L$. Managers maximize the real option's value, $D(\{\nu, \Sigma\}; \Theta)$, where Θ={all model parameters}, $\{\nu,\Sigma\}$={all state variables including current project value}, and Σ has all the state variables except the project value, such as variance. The optimal boundary satisfies the smooth-pasting condition, $\frac{\partial D(\Theta;\{\nu,\Sigma\})}{\partial \nu}|_{(\nu=L^*)} = 0$. Moreover, the real option has a positive value based on the rationality of investors. The value is increasingly monotonic and continuous from assumptions 1 and 2. Thus, the smooth-pasting condition has a unique root and for two different project values (ν_1, ν_2), the condition yields the same optimal boundary. In another words, the optimal exercise policy for two completely similar projects with different income values is unique $(L^*[\Theta; \{\nu_1, \Sigma\}] = L^*[\Theta; \{\nu_2, \Sigma\}])$. $\square$

J.2 Simulation results without the real option to wait

The real option is the sole driver of investment dynamics in the simulations. Without the real option to wait, the simulations show no meaningful relation between investment and its determinants. Table 9 shows the results which are comparable to Table 1. The parameter calibrations of this exercise is based on the same parameters as in Table 8, but the probability of having the real option to wait is zero. Contrary to the model with the real option and earlier studies, correlation between volatility and investment is positive. Another contradictory result is negative correlation between Tobin's Q and the investment. VRP effect is very small and not significantly different from 0. These results imply that the simulations have biased priors towards rejecting H1 without the real option to wait. Hence, the real-option channel is the sole driver behind the relation between asset VRP and investment in this paper.

[**Place Table 9 about here**]

J.3 Regression results with controlling for investment momentum

The inferences about the effect of VRP on investment are robust to the bias of including both lagged dependent variable and fixed-effect dummies. Nickell (1981) argues that correlation between fixed effect and lagged dependent variable biases the estimates when both appear on the regression. However, lagged investment controls for investment momentum and R-squared significantly improves due to adding the new variable. Table 10 reports negative coefficient for VRP, even when the bias exists. Table 11 shows the results where the inferences in empirical data are also robust to this bias.

[**Place Table 10 about here**]

[**Place Table 11 about here**]

Table 9 - OLS Regression results on simulated data without real option to wait: This table is comparable to Table 1. Simulation parameters and details are the same as in Appendix I, except there is no real option to wait, $Pr = 0$. The tables shows the panel regressions on the investment rate (Inv. ratio) on simulated firm-years for 25 years across 1000 firms. Each coefficient represents the mean across 1000 economies. Standard deviation of the coefficients are in parentheses (coefficient distribution p-values are:$*p < 0.1, **p < 0.05, ***p < 0.01$ where they test if the sign for the coefficient' mean, positive or negative, is significant). We estimate the regression $Inv.Ratio_t = a_0 + b_1 \times VRP +$ Control variables $+ u$ with control variables as in Equation 11. Lag variables are lagged for 1 year. Tobin's Q is the ratio of total market value of assets to total capital and size is total market value. Difference in the log size is for 1 year where Log size is the natural log of size. Investment rate is the capital expenditure minus depreciation divided by total capital. Volatility is the instantaneous standard deviation of the cash flows. VRP is the difference between each firms' physical and RA mean reversions.

Lag volatility	0.188***	0.188***	0.006	0.006
	(0.049)	(0.049)	(0.052)	(0.052)
VRP	-	-0.002	-	0.048
	-	(0.002)	-	(0.086)
Lag inv. ratio	0.026	0.026	-	-
	(0.023)	(0.023)	-	-
Diff. Log size	0.911***	0.910***	0.902***	0.902***
	(0.148)	(0.149)	(0.150)	(0.150)
Lag Tobin's Q	-0.127*	-0.127*	-0.156	-0.156
	(0.091)	(0.091)	(0.146)	(0.146)
White noise	0.000	0.000	0.000	0.000
	(0.002)	(0.002)	(0.002)	(0.002)
Intercept	0.160	0.164	0.252	0.145
	(0.132)	(0.133)	(0.217)	(0.307)
firm fixed effect	No	No	yes	yes

Table 10 - OLS Regression results on simulated data: This table is comparable to Table 1. Simulation parameters and details are in Appendix I. The tables shows the panel regressions on the investment rate (Inv. ratio) on simulated firm-years for 25 years across 1000 firms. Each coefficient represents the mean across 1000 economies. Standard deviation of the coefficients are in parentheses (coefficient distribution p-values are:$*p < 0.1, **p < 0.05, ***p < 0.01$ where they test if the sign for the coefficient' mean, positive or negative, is significant). We estimate the regression as in Equation 11, but we include both fixed-effect dummy and lagged dependent variable. Lag variables are lagged for 1 year. Tobin's Q is the ratio of total market value of assets to total capital and size is total market value. Difference in the log size is for 1 year where Log size is the natural log of size. Investment rate is the capital expenditure minus depreciation divided by total capital. Volatility is the instantaneous standard deviation of the cash flows. VRP is the difference between each firms' physical and RA mean reversions. VRP has negative effect on investment (H1).

Model	(1) Inv. ratio	(2) Inv. ratio
Lag volatility	-0.037 (0.172)	-0.037 (0.172)
VRP	- -	-0.248*** (0.237)
Lag inv. ratio	0.029* (0.021)	0.029* (0.021)
Diff. Log size	0.573*** (0.087)	0.573*** (0.087)
Lag Tobin's Q	0.068*** (0.031)	0.068*** (0.031)
White noise	0.000 (0.001)	0.000 (0.001)
Intercept	-0.219*** (0.136)	0.337 (0.419)
firm fixed effect	yes	yes

Table 11 - Bias-robust regression results (1998-2014): This table is comparable to Table 3. The table shows the panel regressions on the investment rate (Inv. ratio) on 19,814 firm-years with both lagged investment and fixed effect controls. We estimate the regression $Inv.Ratio_t = a_0 + b_1 \times VRP_{t-1} + \text{Control variables} + b_2 \times IGdummy_{t-1}VRP_{t-1} + \text{IG dummy} \times \text{Control variables} + \eta_t + \eta_i' + u_t + u_i + u$. Control variables are in Equation 12 with the addition of the lagged investment ratio. Involving lagged dependent variable and fixed effect dummies create panel estimation bias, but inferences on asset VRP effect are robust to this bias. All the variables have the same definition as Table 3. Standard errors are in parentheses (p-values are:$*p < 0.1, **p < 0.05, ***p < 0.01$). VRP has negative effect on investment based on the sign for b_1 (H1). VRP has almost two times stronger negative effect for IG firms (H2) compared to the average firms based on the sign and magnitude for b_2 in Model 6 ($\frac{-0.5\%-0.4\%}{-0.4\%}$).

Model	(1) Inv. ratio	(2) Inv. ratio	(3) Inv. ratio	(4) Inv. ratio	(5) Inv. ratio	(6) Inv. ratio
Lag volatility	-0.0297***	-0.0298***	-0.0306***	-0.0307***	-0.0273***	-0.0275***
	(0.00665)	(0.00663)	(0.00661)	(0.00660)	(0.00755)	(0.00754)
Lag VRP	-0.00448**	-0.00434**	-0.00499**	-0.00478**	-0.00354	-0.00333
	(0.00223)	(0.00219)	(0.00214)	(0.00210)	(0.00220)	(0.00217)
Lag Inv. Ratio	0.235***	0.234***	0.229***	0.227***	0.218***	0.216***
	(0.03180)	(0.03200)	(0.03180)	(0.03200)	(0.03030)	(0.03040)
IG dummy	0.00299	0.00382	0.00339*	0.00419	0.00315	0.00427
	(0.00183)	(0.00239)	(0.00190)	(0.00257)	(0.00215)	(0.00274)
Lag volatility × IG	-	0.00299	-	0.0000701	-	0.00297
	-	(0.00659)	-	(0.00623)	-	(0.00604)
Lag VRP × IG	-	-0.00392	-	-0.00738**	-	-0.00605**
	-	(0.00262)	-	(0.00305)	-	(0.00302)
Lag Inv. Ratio× IG	-	0.172***	-	0.168***	-	0.181***
	-	(0.05540)	-	(0.05510)	-	(0.05980)
firm & year fixed effect dummies	yes	yes	yes	yes	yes	yes
firm & year clustered errors	yes	yes	yes	yes	yes	yes
R^2	11.40%	11.40%	12.30%	12.40%	14.90%	14.90%
BIC	-67485.7	-67467.5	-67676.1	-67642.4	-68233.9	-68182.7
Obs	19,814	19,814	19,814	19,814	19,814	19,814
Other control variables	Intercept	intercept	intercept, Diff. log sales, Tobin's Q	intercept, Diff. log sales, Tobin's Q	intercept, Δlog(sales), Tobin's Q, Profitability, cash ratio	intercept, Δlog(sales), Tobin's Q, Profitability, cash ratio

J.4 Variance risk premium and investment in a larger sample

Similar to the earlier trends reported in Section 3.1, we check the trends to support the hypotheses in a larger data sample without limiting data to option-implied volatility. Figure 8 and Figure 9 show the results that are similar to Figure 5 and Figure 6. All the numbers in the next two figures are the averages of the firm-years for each grade. For all the rated firm-years between 1996 and 2014, we combine annual financial data from merged Compustat-CRSP with 365-day historical equity volatility from Optionmetrics without option-implied volatility. Hence, the sample is larger in Figure 8. We unlever the parameters using the firm's leverage ratio, the ratio of book debt to the market value of equity plus book value of debt. The trends are still puzzling similar to inferences from Figure 5. Since there is no option-implied volatility, we calculate the correlation between historical 365-day equity volatility and historical 30-day S&P-500 volatility as a proxy for VRP. Although the correlation calculation shrinks the sample in Figure 9 compared to Figure 8, the observation about high exposure of the IG firms to systematic volatility compared to the SG firms remains the same. Since the exposure to systematic volatility is high for the IG firms in Figure 9, it seems reasonable that their managers invest conservatively to avoid systematic-volatility shocks and high asset VRP. The results support H1 and H2.

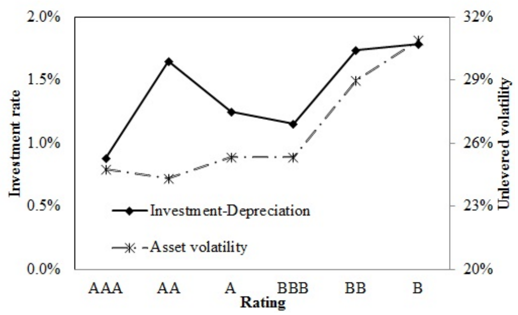
[Place Figure 8 about here]

[Place Figure 9 about here]

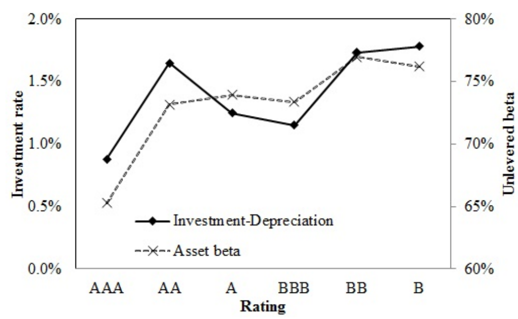
J.5 Statistical comparison of the investment and investment determinants

The trends reported in Figure 5 and Figure 6 to support Hypothesis 2 are also statistically significant. We run the following regressions to compare the averages between IG firms and SG firms:

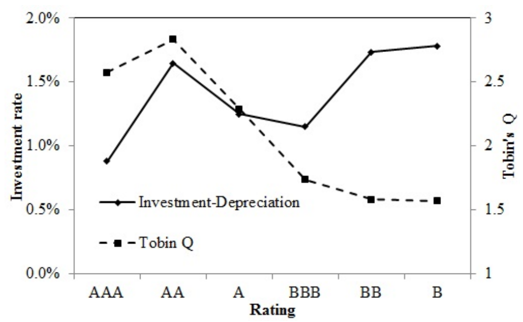
$$X = a_0 + a_1 \times IGdummy + u_t + u_i + u \tag{37}$$



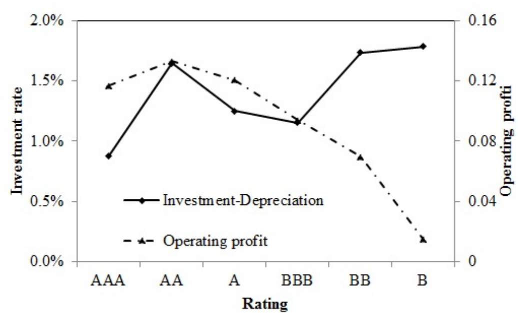
(a) Investment rate and asset volatility



(b) Investment rate and asset beta



(c) Investment rate and Tobin's Q



(d) Investment rate and net profit

Figure 8 The trends of the average net-investment rate in contrast with some of the main determinants of the investment across risk grades.

On all the X-axes, the rating grades (AAA: highest grade, B: lowest grade) are proxy for the risk categories. In each figure, the left Y-axis shows the investment rate. The right Y-axis measures the determinant. Figure 8a shows the average asset return volatility for each category which increases. Figure 8b shows the average asset beta for each category which increases. Figure 8c shows the average Tobin's Q for each category which decreases. Figure 8d shows the average profitability for each category which decreases. The data for the investment rate, asset beta, profitability and Tobin's Q are the average of firm-years in each grade between 1998 and 2014. The asset return volatility is calibrated to the average characteristics for the firm-quarters in each grade between 1996-2011 (borrowed from Lotfaliei (2012)and also are reported by the calibrations in McQuade (2012) and Elkamhi, Ericsson, and Jiang (2011)). De-levered equity volatility for the firm-years in each grade between 1998 and 2014 also yield a similar trend. The trends in the major investment determinants seem counter-intuitive with respect to the observed investment-rate trend.

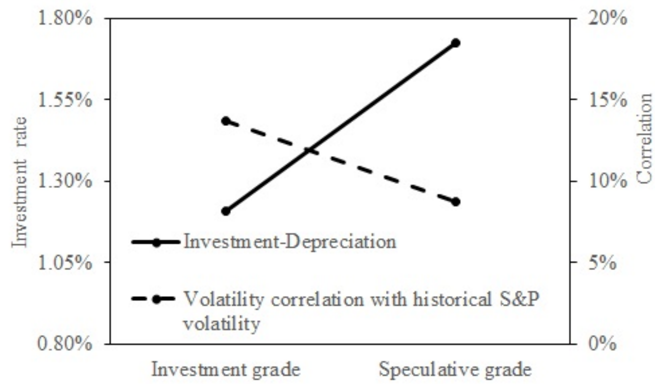


Figure 9 The trends of the average net-investment rate in contrast with the exposure to systematic historical volatility across risk grades.
The figure is comparable to Figure 6b. On the X-axis, the rating grades are proxy for the risk categories. The left Y-axis shows the investment rate. The right Y-axis measures the 30-day correlation between historical 365-day equity volatility and historical 30-day S&P-500 volatility.

where X is the variable for which we compare the mean between IG firms and SG firms, IG dummy is 1 if the company has investment grade during the firm-year, and u_i and u_t control for clustered errors for the firm and time respectively. a_0 represents the mean for variable X and a_1 represents the change in the mean for IG firms. For example, since IG firms invest less than the other rated firms, the sign for a_1 is negative and statistically significant when X is the investment ratio or investment ratio with R&D expenses. Table 12 reports the regression results, which support Hypothesis 2.

[**Place Table 12 about here**]

J.6 Regression coefficients for out-of-sample tests

Table 13 reports all the regression coefficients on data before the break point used to predict the investment ratios for firm-years after the break points. The table is similar to Table 3. The signs for the effect of VRP on the firms' investment is negative and VRP has also large and significant negative effect on IG firms compared to an average firm as expected by hypotheses 1 and 2. However, this regression does not control for time and fixed effects.

Table 12 - Statistical comparison between average investment and investment factors across IG and SG firms: The table supports the trends in Figure 5 and Figure 6. The table shows the estimation of the following regressions: $X = a_0 + a_1 \times IGdummy + u_t + u_i + u$ where X is the variable for which we compare the mean between IG firms and other rated firms, IG dummy is 1 if the company has investment grade during the firm-year, and u_i and u_t control for clustered errors for the firm and time respectively. a_0 represents the mean for variable X and a_1 represents the change in the mean for IG firms. For example, since IG firms invest less than the other rated firms, the expected sign for a_1 is negative when X is the investment ratio. On average, IG firms invest less than SG firms as in models (1) and (2), IG firms are less risky as in models (3) and (4), IG firms are more profitable as in Model (5), IG firms has higher growth potential as in Model (6), and IG firms have higher asset VRP as in models (7) and (8).

Model	(1) Inv. Ratio	(2) Inv. Ratio with R&D	(3) Unlevered volatility	(4) Unlevered beta	(5) Operating profit	(6) Tobin's Q	(7) Asset VRP proxy	(8) Volatility and VIX correlation
IG dummy	-0.0134***	-0.0111***	-0.0343***	-0.000652	0.0463***	0.223**	0.0154	0.101***
p-value	(0.000)	(0.003)	(0.000)	(0.985)	(0.000)	(0.038)	(0.108)	(0.000)
Intercept	0.0221***	0.0332***	0.288***	0.784***	0.0545***	1.667***	1.019***	0.281***
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Table 13 - Regression results: The tables shows the out-of-sample regressions on the investment rate (Inv. ratio) on the firm-years before the break point: $Inv.Ratio_t = a_0 + a_1 \times volatility_{t-1} + b_1 \times VRP_{t-1} + a_2 \times Inv.Ratio_{t-1} + a_3 \times \Delta log(sales)_t + a_4 \times Tobin_{t-1} + a_5 \times CashRatio_t + a_6 \times Profitability + a_7 \times IGdummy_t + IGdummy_{t-1} \times (c_1 \times volatility_{t-1} + b_2 \times VRP_{t-1} + c_2 \times Inv.Ratio_{t-1} + c_3 \times \Delta log(sales)_t + c_4 \times Tobin_{t-1}) + IGdummy_{t-1} \times (c_5 \times CashRatio_t + c_6 \times Profitability_t) + u$. The regression is simple OLS without control for fixed firm and time effects. IG dummy is 1 if the company has investment grade during the firm-year. Lag variables are lagged for 1 year. Difference in the log sales is for 1 year. All the variables are similar to Table 3. Standard errors are in parentheses (p-values are:$*p < 0.1, **p < 0.05, ***p < 0.01$).

Model	(1) Inv. ratio	(2) Inv. ratio	(3) Inv. ratio	(4) Inv. ratio	(5) Inv. ratio	(6) Inv. ratio
Sample break point		01, Jan, 2010			01, Jan, 2013	
Lag volatility	-0.0477*** (0.0045)	-0.0458*** (0.0041)	- -	-0.0353*** (0.0035)	-0.0338*** (0.0032)	- -
Lag VRP	-0.00413* (0.0024)	- -	- -	-0.00310* (0.0018)	- -	- -
Lag inv. ratio	0.555*** (0.0232)	0.555*** (0.0232)	- -	0.574*** (0.0214)	0.575*** (0.0214)	- -
Diff. Log sales	0.00553** (0.0028)	0.00551* (0.0028)	- -	0.00603*** (0.0021)	0.00602*** (0.0021)	- -
Lag Tobin's Q	0.000713** (0.0003)	0.000726** (0.0003)	- -	0.000707** (0.0003)	0.000722** (0.0003)	- -
Lag Cash ratio	0.0130*** (0.0029)	0.0124*** (0.0029)	- -	0.00691*** (0.0023)	0.00648*** (0.0023)	- -
Lag Profitability	0.00607 (0.0045)	0.00662 (0.0045)	- -	0.00803** (0.0040)	0.00851** (0.0039)	- -
IG dummy	-0.00133 (0.0023)	-0.00455** (0.0021)	- -	-0.0011 (0.0020)	-0.00406** (0.0018)	- -
Lag volatility * IG	0.00903 (0.0069)	0.00514 (0.0066)	- -	0.00458 (0.0056)	0.00117 (0.0054)	- -
Lag VRP * IG	-0.00804*** (0.0028)	- -	- -	-0.00677*** (0.0023)	- -	- -
Lag inv ratio * IG	0.218** (0.0904)	0.228** (0.0888)	- -	0.187** (0.0754)	0.196*** (0.0741)	- -
Lag diff. Log sales * IG	0.00549 (0.0057)	0.00556 (0.0057)	- -	0.00636 (0.0050)	0.00695 (0.0050)	- -
Lag Tobin's Q * IG	-0.000589 (0.0010)	-0.00186** (0.0009)	- -	-0.00029 (0.0009)	-0.00128 (0.0008)	- -
Lag Cash ratio * IG	0.0145 (0.0132)	0.0114 (0.0136)	- -	0.0088 (0.0087)	0.00834 (0.0087)	- -
Lag Profitability * IG	-0.0242 (0.0205)	-0.0283 (0.0199)	- -	-0.0166 (0.0180)	-0.0199 (0.0175)	- -
Intercept	0.0264*** (0.0040)	0.0212*** (0.0022)	0.00789*** (0.0008)	0.0209*** (0.0032)	0.0169*** (0.0018)	0.00875*** (0.0006)
R^2	0.348	0.348	0	0.357	0.357	0